INTRODUCTION

This is a topical trial questions book designed to tackle major topics in the current syllabus.

The questions are graded in each topic and the main objective is to present the material in a manner for easy comprehension and understanding.

The book consists of Review sections in a number of topics which help a learner to recall several formulars where need be and there exists examples in some topics where need be in order to bring out the meaning clearly.

The main reason for the designing of this book is to provide enough questions for the students' practice in order to perfect in the subject and also provide confidence in the students after being exposed to a number of questions in this book.

It is hoped that the book will be found usefull for students' revision questions at O'level.

Any suggestions for improvement of this book are most welcomed.

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CONTENTS

FRACTIONS AND OPERATIONS

RECALL

◊ A fraction is referred to as part of a whole quantity. It represented as $\frac{\text{numerator}}{\text{denominator}}$.

◊ **Proper fraction** is a fraction whose numerator is less than the denominator

◊ **Improper fraction** is a fraction whose numerator is greater than the denominator

◊ **Mixed fraction** is a fraction containing a whole number and a proper fraction.

◊ **Operations on fractions**; This involves addition, subtraction, division and multiplication. But with addition and subtraction, if the denominators are different, we calculate their **LCM** and then proceed normally. On the other hand, if the denominators are the same, we add the numerators directly.

◊ When dividing two fractions, we find the reciprocal of the second fraction and then multiply them

◊ When multiplying two or more fractions, we multiply the numerators together and multiply the denominators also together.

◊ Compound operations on fraction; here we consider cases of simplifying fractions in which brackets and other arithmetic operations are involved. This process is governed by law called BODMAS

◊ Converting recurring decimals to fractions eg 0.66666666……….to fractions

Operations

◊ Operations as a topic is represented by several symbols connecting an equation. The operations may include; ↓, ↑, *, @, Δ, ∇ ,∅,φ, etc. for example; If $a*b = a^2 - b^2$. Find 3*2. This implies that 3 stands for a and 2 stand for b. Thereafter substitute the values into the equation;

$3*2 = 3^2 - 2^2$

$9 - 4 = 5$

Therefore, 3*2 = 5

TRIAL QUESTIONS

1. Convert 5.2727………….. to fractions. **[Ans: $\frac{58}{11}$]**
2. Simplify $\frac{2}{3} + \frac{1}{5}$ of $\frac{1}{2} \div 1.25$ **[Ans: $\frac{56}{75}$]**
3. Solve for a in $\frac{a+8}{2} - \frac{3(a+10)}{5} = 1$ **[Ans: $a = {}^-30$]**
4. A bottle contained $6\frac{2}{5}$ litres of milk. Berochan took $\frac{3}{8}$ of the milk and Oola took $\frac{3}{4}$ of the remainder. How much milk remained in the bottle. **[Ans: 1 litre]**
5. Express 0.245…… in the simplified form $\frac{a}{b}$. Hence find $b - a$ **[Ans: $\frac{245}{999}$, $b - a = 754$]**
6. Given that X * Y = $X^2 + Y$. Find 4*3 **[Ans: 19]**
7. Express 3.36 as an improper fraction in it's simplest form. **[Ans: $\frac{84}{25}$]**
8. If $a * b = \frac{a}{b} + \frac{b}{a}$. evaluate $\frac{1}{2} * \frac{3}{2}$ **[Ans: $\frac{10}{3}$]**
9. Work out $(4\frac{1}{2} - 3\frac{2}{3}) \div 1\frac{2}{3}$ **[Ans: $\frac{1}{2}$]**
10. Simplify $5\frac{1}{6} - 3\frac{2}{3} + 6\frac{7}{12}$ **[Ans: $\frac{{}^-61}{12}$]**
11. Write 0.4545… in the form $\frac{a}{b}$ in its simplest form, hence find the value of $a - b$ **[Ans: $\frac{5}{11}$, $a - b = {}^-6$]**
12. (a)Simplify : $\frac{9}{16} \times 2\frac{2}{3} - \frac{9}{6} \div \frac{1}{8}$ **[Ans: $\frac{{}^-21}{2}$]**

    (b) Evaluate $2\frac{10}{11} \times 4\frac{1}{6} \div 1\frac{1}{5}$ **[Ans: $10\frac{10}{99}$]**

    (c) Given that $m * n = 2n - 3m$ find the value of $(2 * {}^-1) + ({}^-3 * 2)$ **[Ans: 5]**

13. Simplify without using a calculator. $\frac{^-3-(^-12)\times 4-(^-20)}{^-6\times 6\div 3+(^-6)}$ **[Ans: $\frac{^-65}{18}$]**

14. Solve for x if; $\frac{x-2}{3}=\frac{x-3}{3}+\frac{x+3}{8}$. **[Ans: $x=\frac{^-1}{3}$]**

15. Simplify; $\frac{1\frac{1}{2}-[8\frac{1}{3}\div 2\frac{1}{2}]}{1\frac{1}{5}\text{ of }[1\frac{1}{4}+1\frac{2}{3}]}$ **[Ans: $\frac{^-11}{21}$]**

16. Given that $x * y=\frac{x+y}{x-y}$, find the value of $(5 * 3)* - 2$. **[Ans: $\frac{1}{3}$]**

17. (a) Simplify; $\frac{1\frac{4}{5}\text{ of }\frac{25}{18}\div 1\frac{2}{3}\times 24}{2\frac{1}{3}-\frac{1}{4}\text{ of }12\div\frac{5}{3}}$ **[Ans: $\frac{135}{2}$ or $67\frac{1}{2}$]**

(b) Solve for x in the equation; $\frac{(x+3)}{4}-\frac{3(4-2x)}{4}=3\frac{7}{10}$ **[Ans: $x = 3.4$]**

18. (a) Express 0.83333 … as a fraction in its simplest form. **[Ans: $\frac{5}{6}$]**

(b) Given that $x*y=\frac{x^2+y^2}{10y}$, evaluate $7 * (4 * {}^-8)$. **[Ans: $^-5$]**

(c) Evaluate; $6\frac{1}{4}\div\frac{8}{13}\times\frac{8}{13}$ of $\frac{26}{52}$ **[Ans: $\frac{25}{8}$]**

(d) Express the recurring decimal as a fraction 0.1515… **[Ans: $\frac{5}{33}$]**

19. (a) Given that $a * b = a^2 - b^2$, find the value of x in $x * \sqrt{3} = 7 * 4$. **[Ans: $x = 6$]**

(b) Evaluate; $\frac{2\frac{3}{4}-1\frac{7}{8}}{2\frac{3}{4}+1\frac{5}{8}}$ (c) Evaluate; $\frac{7\frac{1}{8}\div 1\frac{2}{3}}{\frac{1}{4}+9\frac{1}{2}}$ (d) Simplify; $\frac{3-\frac{1}{2}\text{ of }\frac{1}{3}}{7\frac{1}{3}\div 3\frac{2}{3}}$ **[Ans: b) $\frac{1}{5}$, c) $\frac{57}{130}$, d) $\frac{17}{12}$]**

20. (a) Solve the equation; $\frac{3x-1}{5}-\frac{9-2x}{3}=1$ **[Ans: $x=\frac{63}{19}$]**

(b) Evaluate; $3 * (1 * 4)$ if $a * b = a^2 + \sqrt{ab}$. **[Ans: 12]**

(c) Convert 0.891891891....into a fraction in its simplest form. **[Ans: $\frac{33}{37}$]**

21. (a) Evaluate; $\left(\frac{1\frac{1}{2}+3\frac{1}{6}}{4\frac{1}{3}-3\frac{2}{5}}\right)\div 1\frac{2}{3}$ **[Ans: 3]**

(b) If $P * R = \frac{P^2-R^2}{PR}$, find $(4 * 2) *1.5$ **[Ans: 0]**

(c) The operation $a \wedge b$ is defined as $\frac{a(8+b-3a)}{a+b}$.

Find; (i) $2 \wedge 1$ **[Ans: 2]**

(ii) $3 \wedge (2 \wedge 1)$ **[Ans: $\frac{3}{5}$]**

d) Evaluate; $\frac{\frac{3}{4}+1\frac{5}{7}\div 1\frac{4}{7}\text{ of }2\frac{1}{3}}{[1\frac{3}{7}-\frac{5}{8}]\times\frac{2}{3}}$ **[Ans: $2\frac{3}{11}$]**

22. (a) Solve for t in the equation; $\frac{t}{t-9}=13+\frac{t}{t-9}$. **[Ans: $t = 9$]**

(b) Given that $a * b=\frac{a^2+b}{10b}$, find

i) $4 * {}^-8$ **[Ans: $^-0.1$]**

ii) $7 * (4 * {}^-8)$ **[Ans: $^-48.9$]**

23. (a) Express 0.277777 ….. as a fraction in its simplest form. **[Ans: $\frac{5}{18}$]**

(b) Evaluate; $\dfrac{3\frac{1}{2} - 1\frac{5}{6} \times \frac{3}{11}}{1\frac{3}{4} + 7\frac{2}{3} \div 3\frac{5}{6}}$ **[Ans: $\frac{4}{5}$]**

(c) Solve the equation; $\frac{(x+1)}{3} - \frac{x}{6} = \frac{(x-2)}{4}$ **[Ans: $x = 10$]**

24. (a) Evaluate; $2\frac{1}{2} \div \dfrac{4\frac{1}{3} - 2\frac{1}{4}}{4\frac{1}{6}}$ (b) Evaluate; $\dfrac{1\frac{1}{6} - 3\frac{1}{2}}{4\frac{1}{3} - 1} \div \frac{1}{5}$ **[Ans: (a) 5, (b) $\frac{^-7}{2}$]**

25. (a) Evaluate; $\dfrac{1\frac{1}{5} + 4\frac{1}{2} \div 1\frac{1}{2}}{3\frac{3}{5} - 2\frac{2}{5} \times 1\frac{1}{4}}$ **[Ans: 7]**

(b) If $a\Delta b$ means $2a + 3b$ and $a * b$ means $5a + 4b$ evaluate the expression $2\Delta(4 * 3)$. **[Ans: 100]**

(c) A ball is dropped from a height of 81cm. After each bounce it rebound two-thirds of the distance it fell. How far does the ball fall in the fifth fall? **[Ans: 270 *cm*]**

(d) Given that, $x \,\Delta\, y = x^2 - 6y^2$, evaluate $(8\Delta - 3)\Delta 4$. **[Ans: 4]**

26. (a) The operations * and ^ are defined as $m * n = am + n$ and m^$n = m^2 - n$.

Find the value of a if $^-4$^$(2 * 3) = 9$ **[Ans: $a = 2$]**

(b) Simplify $\dfrac{\left(\frac{^-1}{2}\right) \div \left(\frac{^-2}{3}\right) \times 8 - 4\frac{1}{2}}{\frac{3}{4} - \left(2\frac{3}{4}\right) \div \frac{11}{8}}$ **[Ans: $\frac{^-6}{5}$]** (c) . Simplify: $\dfrac{5\frac{1}{4} \div 4\frac{1}{5}}{2\frac{1}{2}} + 3\frac{1}{4}$ **[Ans: $\frac{15}{4}$]**

(d) Given that $a\Psi b = \frac{b^2 - a^2}{a^2 + b^2}$, find the values of (i) $1 \,\Psi\, ^-1$ (ii) $(1 \,\Psi\, ^-1)\Psi\, ^-4$ **[Ans: (i) 0, (ii) 1]**

(e) Without using a calculator , simplify $\frac{14}{3a}$ of $1\frac{1}{6} \div \frac{4}{9a^2} \times 1\frac{4}{5}$ **[Ans: 14.7*a*]**

(f) If $p*q = p^2 - 3q$ find (i) 5*3 (ii) x such that $[(x * 2) * 3] = 7$ **[Ans: (i) 16, (ii) $x = \sqrt{10}$ or $x = \sqrt{2}$]**

27. A car consumes one litre of petrol for every $\frac{2}{5}$ *km* of distance covered. How many litres of fuel are needed to cover a distance of 55 *km* by the same car? **[Ans: 137.5 litres]**

28. A boy ate one fifth of a pineapple and the sister ate three quarters of what was left.

(a)What portion of the pineapple was eaten all together?. **[Ans: $\frac{4}{5}$]**

(b)What fraction of the pineapple was left? **[Ans: $\frac{1}{5}$]**

29. Namulondo spent $\frac{2}{5}$ of her money on sugar, $\frac{3}{8}$ on pens and $\frac{1}{5}$ on pencils. She has shs3000 left in her pocket. How much money did she have before she bought the items? **[Ans: shs 120,000]**

30. One day Jesca decided to travel from Kampala to Tororo. He covered $\frac{3}{5}$ of the distance on foot and the remaining distance by taxi. He was left with 30 km to complete the journey. Find the total distance between Kampala and Tororo. **[Ans: 75 *km*]**

31. In a refrigerator, $\frac{8}{5}$ of the items stored are drinks.Items are categorized into; drinks and eats. If all the eats as well as $\frac{3}{4}$ of the drinks are bad, what fraction of the items are good? **[Ans: $\frac{2}{5}$]**

LCM, BASES, HCF

TRIAL QUESTIONS

1. Muvule trees are planted in Busoga on both sides of the road at 30 *m* apart on one side of the road and 45 *m* apart on the other side. At one end of the road, two trees are directly opposite each other. After what distance, measured from that end would a pair of trees be directly opposite each other? **[Ans: 90 *m*]**
2. (a) Find the L.C.M of 18, 45 and 42 **[Ans: 630]**
   (b) Find the LCM of 42, 48 and 36 **[Ans: 1008]**
3. Which value of n gives $304_n = 117_{\text{eight}}$ **[Ans: $n = 5$]**
   (b) Given that $202_n = 57_{\text{nine}}$ find the base that n represents **[Ans: $n = 5$]**
4. (a) Find the LCM of 9, 10, 12 and 15 **[Ans: 180]**
   (b)Find the L.C.M of $3a^2b^2c$ and $6ab^2c^3$ **[Ans: $6ab^2c^3$]**
5. Given that $72x = 213_{\text{five}}$ find the value of x. **[Ans: $x = 8$]**
6. Given that $3t5_{\text{eight}} = 245_{\text{ten}}$. Find the value of t. **[Ans: $t = 6$]**
7. (a) Convert 200_{five} to actal base. **[Ans: 50_{ten}]**
   (b) Find the value of base x in the equation; $305_x + 45_x = 353_x$ **[Ans: $x = 7$]**
8. Find the LCM of the following numbers:
   (i) 88, 96, and 126 **[Ans: 22176]** (ii) 56 and 84 **[Ans: 168]**
   (iii) 112, 152, 160, 172 **[Ans: 915040]** (iv) 144, 72, 60 and 54 **[Ans: 4320]**
9. The bell for changing lessons for O'level at Luwero high school rings after 40 minutes while that for A'level rings after 60 minutes. Lessons for both 'O' and 'A' level begin at 7:20 *am* with the sounding of the bells. Find the time when the two bells ring together again. **[Ans: 9:20 *am*]**
10. Three bells ring at intervals of 20, 30, 40 seconds. If they begin by ringing together, after what length of time will they ring together again? **[Ans: 120 seconds]**
11. Makanga, Zahara and Jjingo are running a 10,000 metres race. Makanga completes his first lap after 120 seconds, Zahara completes her first lap after 130 seconds while Jjingo completes his after 142 seconds. When will they next be all at the starting point together? **[Ans: 110760 seconds]**
12. A family of Mr & Mrs.Kirya and their daughter was promised some money from a their family friend. Find the smallest sum of money which can be given to a family so that a child, mother and father can get shs 7500, shs 9000 and shs12000 respectively. **[Ans: 180,000]**

HCF

13. A certain number of oranges can be divided into equal trips each containing 10, 15 or 25 oranges. Find the smallest number of oranges for which this is possible. **[Ans: 5]**
14. Square tiles are used to cover an area measuring 16.5 *m* by 12.75 *m*. If the tiles are all alike and only whole ones are used, what is the greatest size they can be? How many are there?
    [Ans: 0.5625 m^2, 374 litres]
15. (a) Find the H.C.F of 18, 45 and 42 **[Ans: 3]**
    (b) Find the HCF of 42,48 and 36 **[Ans: 6]**
16. Equal squares, as large as possible, are ruled off on a rectangular board 54 by 78 *cm*.
    (i) Find the size of each square. **[Ans: 6×6 *cm* or 36 cm^2]**
    (ii) How many squares are there? **[Ans: 117]**

17. If the product of two numbers is 1260 and their HCF is 6, find their LCM
18. Find the length of the shortest piece of string that can be cut into equal lengths, each of either 28 *cm*, 35 *cm* or 42 *cm*.
19. A rectangular piece of room measuring 10 *m* long by 8 *m* wide is to be covered with square tiles of side x *cm* each.
    (a) Find the value of x **[Ans: $x = 2$ *m*]**
    (b) Find how many tiles are needed if only whole ones are used? **[Ans: 20 tiles]**
20. Find the HCF of; (i)136, 142, and 156
    (ii) 112, 144 and 210 (iii) 72, 108, 180 and 28
21. The sitting room of a house measures 1440 *cm* by 1640 *cm*. it's floor is to be tiled using square tiles which are as large as possible.
    (a) Find the size of these square tiles. **[Ans: 40 × 40 or 1600 cm^2]**
    (b) How many square tiles are required for this purpose?. **[Ans: 1476 tiles]**
22. Three pieces of timber of length 164 *m*, 286 *m* and 358 are to be cut into smaller pieces of the same length. Find the greatest possible length which can be cut without leaving any remainder. **[Ans: 2 *cm*]**
23. (a) Find the HCF of a^3b^4c, $a^2b^3c^2$ and $a^4b^2c^3$. **[Ans: a^2b^2c]**
    (b) The L.C.M of 24, 36 and 60 is x and their G.C.D is y, find the ratio $y : x$. **[Ans: 1:60]**

CONSTRUCTION

RECALL

Before attempting any question in construction, you must;

◊ Have a pair of compasses, a set, and a ruler

◊ Know how to construct relevant angles

◊ Be able to bisect and angles, lines and know the relevance of the respective process.

◊ Be able to construct triangles etc

◊ Be able to drop a perpendicular from a point which is not on the line onto the line.

◊ Be able to inscribe and circumscribe a triangle and any figure where necessary

◊ Recall the formula for finding the area of a triangle and a circle.

TRIAL QUESTIONS

1. (a) Construct triangle ABC with AB = 8 *cm*, BC = 5 *cm* $\angle ABC = 75^0$, measure AC and $\angle BAC$.
   (b) Draw a circle circumscribing triangle ABC and find its radius
2. Using a ruler and pair of compasses only construct triangle PQR in which $\angle RPQ = 60^0$ $\angle PQR = 45^0$ PQ = 6 *cm*. Measure the length
   (i) $\overline{PR}$ (ii) $\overline{QR}$
   Construct a circumcircle through PQ and R. Measure its radius hence find the area of the circle.
3. (a) Using a ruler and pair of compasses only construct triangle ABC in which $\angle BAC = 60^0$ $\overline{AB} = 6.2$ *cm* $\overline{AC} = 7.1$ *cm*
   Measure; (i) $\overline{BC}$
   (ii) The radius of the circumcircle you have constructed hence find its area.
4. (a) Construct triangle ABC in which AB = 5 *cm* BC = 9 *cm* and $\angle ABC = 105^0$ using a ruler and pair of compasses only
   (i) Draw a circle passing through A , B and C and measure its radius
   (ii) Find the area of the circle.
5. Using a ruler and a pair of compasses only
   (a) Construct a triangle ABC in which AB = 6 *cm*, BC = 8 *cm* and angle ABC = 60^0.
   (b) Inscribe a circle in the triangle ABC and measure its radius.
   (c) Construct a perpendicular from vertex A to meet BC at R. Measure AR.
   (d) Calculate the area outside the circle but inside the triangle correct to 3 significant figures.
6. (a) Using a ruler and a pair of compasses only, construct a rhombus ABCD in which AB = BC = 5 *cm* and angle ABC = 120^0.
   (b) Measure angle DAB.
   (c) Construct perpendicular bisectors to sides AB and BC to meet at O.
   (d) Construct a circle that passes through vertices A, B and C.
7. (a) Using a ruler and a pair of compasses only construct a trapezium ABCD in which AB is parallel to DC and AB = 6 *cm*, AD = 4 *cm* DC = 3 *cm* and angle DAB = 60^0.
   (b) Drop a perpendicular from D to meat AB at M. Measure DM.
   (c) Calculate the area and perimeter of the trapezium ABCD.

8. Using a ruler, pencil and a pair of compasses only;

   (a) Construct a triangle ABC, where AB = 8.0 *cm* and C is 5.4 *cm* from A and 6.5 *cm* from B.

   (b) D is a point on AC produced such that AD = 7.4 *cm* and E is 6.2 *cm* from D on the same side of AD as B, such that angle A DE = 135^0

   (c) Draw a circle to circumscribe points A, B and E. Measure

   (i) $\overline{AE}$

   (ii) The radius of the circle

9. (a) using a ruler and a pair of compasses only, construct a parallegram ABCD such that AB = 9 *cm*, BC = 8 *cm*, angle BAC = 45^0 and angle ABC = 135^0.

   (b) Bisect angles ABC and BAD and let the bisectors meet at X. Measure AX.

   (c) Construct a circle that passes through points A, B and X. Measure the radius of the circle and hence calculate its circumference.

10. (a) Using a ruler and compass only, construct ΔPQR such that PQ = 8 *cm*, QR = 6 *cm* and $\angle QPR = 45^0$.

    (b) Measure $\overline{PR}$ and $\angle PQR$

    (c) Draw a circle that passes through P, Q and R.

    (d) Calculate the area of;

    (i) the circle

    (ii) the triangle PQR

11. Using a ruler, pencil and pair of compasses ONLY.

    (a) Construct the quadrilateral ABCD in which AB = 6 *cm*, BC = 5 *cm*, CD = 7 *cm*, angle ABC = 105^0 and angle BCD = 120^0.

    (b) Construct an in circle of triangle ACD with centre O.

    (c) Measure; (i) the length OA

    (ii) the radius of the circle.

12. Construct triangle ABC such that $\overline{AB}$ = 4 *cm*, $\overline{BC}$ = 3.6 *cm* and $\overline{AC}$ = 3.8 *cm* . Construct perpendicular bisectors of lines $\overline{AB}$ and $\overline{AC}$ such that they intersect at point P. Measure line AP. Draw a circumcircle of triangle ABC.

13. Using a ruler and a pair of compasses only, construct a quadrilateral ABCD in which AB = 5 *cm*, BC= 6 *cm*, CD = 4 *cm* and angle ABC = 75^0 and angle BCD = 120^0. Construct a circle passing through points A, C and D.

    (a) Measure

    (i) AD

    (ii) the distance from the centre of the circle to point B

    (b) Find the area of the circle

14. Using a ruler, pencil and a pair of compasses only, construct a quadrilateral PQRS such that PQ = 5 *cm*, QR = 7 *cm*, angle SPQ = angle PQR = 90^0 and angle QRS = 60^0.

    (b) Construct a circle circumscribing triangle SQR. Hence measure;

    (i) length RS and PS.

    (ii) the angle PSR

    (iii) the distance from the centre, O, of the circle to R.

    (c) Calculate the area of the circle.

15. Using a ruler and pair of compass only. Construct triangle ABC such that $\overline{BC} = 4.8$ *cm* and angles BAC $= 30^0$, and ABC $= 90^0$. D is a point on $\overline{BC}$ produced 2.7 *cm* away from $\overline{AB}$. Construct angle BDE $= 45^0$ with $\overline{DE} = 10.1$ *cm*. Join points A to D and B to E. Construct a circle circumscribing triangle ACD such that it also passes through the point E. Measure;

(i) length AB and BE

(ii) angle ADC

(iii).the radius of the circle

16. (a) On a squared paper, plot points P(2.0, 3.5), Q(7.0, 3.5) and R(4.5, 6.5). Use a scale of 1 *cm* to represent 0.5 units on both axes. Join the points to form triangle PQR.

(b) Using a ruler and a pair of compasses only construct a circle circumscribing triangle PQR. State the coordinates of the centre, C, of the circle.

(c) Measure its radius and angle PCQ. Hence calculate area of the minor sector PCQ.

CO-ODINATE GEOMETRY

RECALL

- ◊ $y = mx + c$ is the general form of an equation of line, where m is the gradient of a line and C, the intercept.
- ◊ Parallel lines have the same gradient $M_1 = M_2$
- ◊ For perpendicular lines, the product of their gradient is $^-1$
- ◊ $M_1M_2 = {}^-1$
- ◊ To find the point of intersection of two lines, we solve the two equations simultaneously to find the values of x and y.
- ◊ Mid point of a line $= \left(\frac{x_1 + x_2}{2}, \frac{y_2 + y_1}{2}\right)$
- ◊ Gradient of a line, $m = \frac{y_2 - y_1}{x_2 - x_1}$
- ◊ Distance between two points $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

TRIAL QUESTIONS

1. Find the equation of a line which passes through point (4,3) and parallel to the line $y = 2x + 4$ **[Ans: $y = 2x - 5$]**
2. Find the equation of a line that passes through point (2,5) and is parallel to the line $y = 3x - 15$. **[Ans: $y = 3x - 1$]**
3. (a) Determine the equation of line that passes through points (2, 1) and (3, 3) **[Ans: $y = 2x - 3$]**

   (b) Two lines L_A and L_B are parallel. Line L_A passes through points (4, 2) and ($^-3$, 1). Determine the equation of line L_B that passes through point (7, 0). **[Ans: $y = \frac{1}{7}x - 1$]**
4. The line that passes through point A(m, 4) and B(4, m) is parallel to the line whose equation is $y = 2x - 4$. Find the value of m. **[Ans: $m = 4$]**
5. A line through (2, 3) and ($^-1$, 2) crosses at right angle, a line $ax + by = 11$ at point (3, 2). Find the values of a and b. **[Ans: $a = 3, b = 1$]**
6. Determine the equation of the straight line passing through the point (2, 5) and perpendicular to the line joining the point A($^-3$, 3) to point B(6, 9) **[Ans: $y = \frac{^-3}{2}x + 8$]**
7. (a) Find the equation of the line passing through (5, 4) and perpendicular to the line $2y = 3x - 7$ **[Ans: $3y = {}^-2x + 22$]**

   (b) Find the equation of a straight line which passes through the point (0, 5) and is perpendicular to the line $y + 3x = 7$. **[Ans: $3y = {}^-x + 5$]**
8. (a) Find the equation of a straight line that goes through (1, 7) and (4, 9). **[Ans: $3y = 2x + 19$]**

   (b) Find the equation of a straight line whose gradient is $\frac{^-1}{2}$ and passes through the point (4, 5). **[Ans: $y = \frac{1}{7}x + 7$]**

   (c) Find the equation of a straight line which passes through (1, 3) and is perpendicular to the line $2y - 3x = 7$. **[Ans: $3y = {}^-2x + 1$]**
9. (a) Find the equation of a line that passes through the point A(4, 0) and is parallel to the line whose equation is $y = 2x + 4$. **[Ans: $y = 2x - 8$]**

   (b) If A(2, 9) and B($^-1$, 5) are end points of a line segment $\overline{AB}$, find;

   (i) the coordinates of M, the midpoint of line segment $\overline{AB}$. **[Ans: $(\frac{1}{2}, 7)$]**

(ii) the distance of line $\overline{AB}$. **[Ans: 5 units]**

(c) Find the coordinates of the y-intercept and x-intercept of the line whose equation is $y - 4x - 6 = 0$. **[Ans: y intercept (0, 6), x-intercept ($\frac{^-3}{2}$, 0)]**

10. (a) Find the equation of a line that joins A(1, 5) and B($^-2$, $^-1$). **[Ans: $y = 2x + 3$]**

(b) Determine the equation of a straight line which passes through the point (3, 4) and is parallel to the line $y + 3x - 5 = 0$. **[Ans: $y = {^-3}x + 13$]**

11. (a) For the equation $3y - 4x + 9 = 0$, find;

(i) the gradient of the equation. **[Ans: $\frac{4}{3}$]**

(ii) the coordinates of the point where the line in (i) cuts the y-axis. **[Ans: 0, $^-3$]**

(b) Find the equation of a straight line passing through the point (2, 3) and is perpendicular to the line $2y - 4x - 7 = 0$. **[Ans: $2y = {^-x} + 4$]**

12. (a) The line through points A(a, 2) and B(3, 6) is parallel to the line $y = 4x - 5$. Find the value of a. **[Ans: $a = 2$]**

(b) Find the equation of a line passing through a point A(2, 0) and perpendicular to the line joining the points B($^-10$, 3) and C(6, $^-9$). **[Ans: $3y - 4x + 8 = 0$]**

13. The points ($^-1$, q) and (r, 2) lie on the line $y = 2 - x$. Find the values of q and r. **[Ans: $q = 3, r = 0$]**

14. A line is given by the equation $45 - 15x + 3y = 0$. Find the coordinates of the x-intercept. **[Ans: (3, 0]**

15. Two points P(5, 2) and Q(2, 4) are in a plane. Find;

(a) Coordinates of **M** the mid point of $\overline{PQ}$ **[Ans: $\frac{7}{2}$, 3]**

(b) $|\overline{OM}|$ where O is the origin. **[Ans: 4.61 units]**

16. Given the curve $y = 2x^2 + 3x$ and the line $y = 5 + 4$, determine the coordinates of the points of intersection of the curve and the line. **[Ans: ($^-1$, $^-1$), (2, 14)]**

17. (a) Find the equation of a line which is a perpendicular bisector of the line passing through the points P($^-2$, 3) and Q(6, 1). **[Ans: $y = 4x - 6$]**

(b) Find the equation of a straight line which passes through A($^-2$, 6) and B($^-5$, 4) and hence determined the coordinates of the point Q where this line cuts the x-axis. **[Ans: $3y = {^-2x} + 22$, Q (11, 0)]**

18. (a) Find the equation of a straight line through (2, 3) and is parallel to the line $y = \frac{1}{2}x - 7$. **[Ans: $y = \frac{1}{2}x + 2$]**

(b) Find the equation of a line through the origin and perpendicular to the line joining the points P(6, 8) and R(3, 5). **[Ans: $y = {^-x}$]**

19. (a) Find the equation of a line , in the form $y = mx + c$, parallel to $4y + 2x = 5$ and passing through the point(1, $^-3$). **[Ans: $2y + x - 7 = 0$]**

(b) The line $3x - 5y - 6 = 0$ passes through point (p, 0). Determine it's

(i) gradient **[Ans: $\frac{3}{5}$]**

(ii) value of p **[Ans: $p = 2$]**

20. Line M passes through points (8, $^-1$) and ($^-2$, 12). Line N is $2y - x = 2$. Determine;

(a) the equation of line M. **[Ans: $10y + 13x - 94 = 0$]**

(b) the co-ordinates of point of intersection for lines M and N. **[Ans: ($\frac{14}{3}$, $\frac{10}{3}$)]**

(c) the x – intercepts for the lines M and N. **[Ans: ($\frac{94}{13}$, 0) and ($^-2$, 0)]**

(d) the area enclosed between lines M and N and the x – axis.

21. Find the equation of a line whose y - intercept is 5 and perpendicular to the line whose equation is $y - 2x = 3$. **[Ans: $y = \frac{^-1}{2}x + 5$]**

22. (a) Find the equation of a perpendicular bisector of a line passing through (2, 4) and (2, 6)

(b) find the equation of a line whose x - intercept is $^{-}4$ and y − intercept is 5, hence state it's gradient.

[Ans: $y = \frac{^{-}5}{4}x + 5$, Gradient = $\frac{^{-}5}{4}$]

23. Find the lengths of the line joining the following pair of points.

(a) ($^{-}4$, $^{-}1$) and (4, 1) (c) ($^{-}2$, 5) and ($^{-}2$, 7)

(b) ($^{-}1$, $^{-}3$) and ($^{-}9$, $^{-}9$) **[Ans: (a) $\sqrt{68}$ units (b) 10 units (c) 2 units]**

24. (a) Find the distance of the point A($^{-}7$, 24) from the origin.

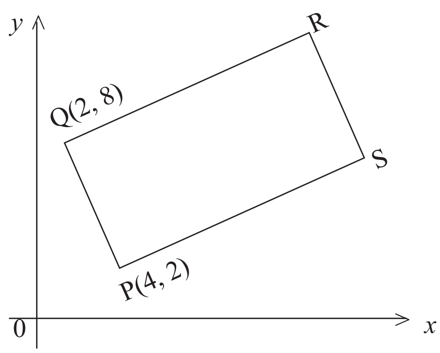
(b) A(4, 7) and B(x, y) are the end-points of the straigh line AB and C(5, 6) is its mid-point. Find the values of x and y. **[Ans: $x = 6, y = 5$]**

25. (a) A(0, 0), B(5, 1), C(8, 4) and D(3, 3) are the vertices of a parallelogram. Find the length of the diagonal AC. **[Ans: $\sqrt{80}$ units]**

(b) Show that the co-ordinates of the mid-points of the diagonal AC and BD are the same.

26. P is (2,5) and O is the origin. The line $y = mx + c$ is parallelto OP. What is m? **[Ans: $m = \frac{5}{2}$]**

27. In the figure PQRS is a rectangle and P and Q are the points (4, 2) and (2, 8) respectively. Given that the equation of line PR is $y = x - 2$, find;



(i) the equation of QR. **[Ans: $3y - x = 22$]**

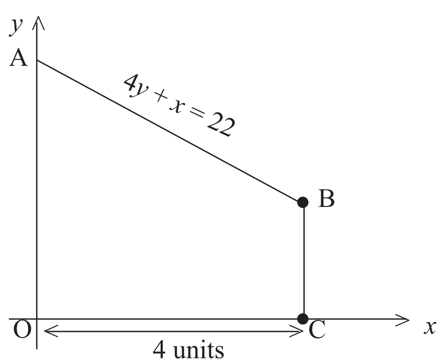
(ii) the coordinates of R **[Ans: 14, 12]**

(iii) the co-ordinates of S. **[Ans: 16, 6]**

(iv) the area of the rectangle PQRS **[Ans: 80 sq.units]**

28 (a) The points ($^{-}1$, q) and (r, 2) lie on the line $y = 2 - x$. Find the values of q and r. **[Ans: $q = 3, r = 0$]**

(b) In the figure below, C is 4 units from O, the equation of the line AB is $4y + x = 22$. Find the area of OABC.



[Ans: 10 sq.units]

BUSINESS MATHEMATICS

RECALL

- ◊ Profit = selling price – cost price
- ◊ Percentage profit = $\frac{\text{profit}}{\text{cost price}} \times 100$
- ◊ Loss = cost price − selling price
- ◊ Taxable income = Gross income − total allowances
- ◊ Income tax is obtained basing on taxable income using given rates as per question. Rates are given in percentage. The taxable income gives the limit for taxation on the individuals income.
- ◊ Expressing income tax as percentage of gross income; $\frac{\text{income tax}}{\text{gross income}} \times 100$
- ◊ Simple interest (I) = $\frac{PRT}{100}$, but also, interest(I) = Amount(A) − Principle(P)
- ◊ Compound interest: $A = P(1 + \frac{r}{100})^n$, A → amount, P→ principle, r → rate, n → number of years.
- ◊ Total hire purchase = Deposit + Total installment.
- ◊ Taxable income = Gross income − Total allowances
- ◊ Gross income = taxable income + total allowances.
- ◊ Net income = Gross income − income tax.

TRIAL QUESTIONS

1. (a) If the exchange rate for shillings to dollars is \$1 = 1675/=, how many dollars does Mukasa get if he has 418, 750/=? **[Ans: \$250]**

   (b) Find the principal which gains shs 30,000 as simple interest in 12 months at a rate of 15% per half a year. **[Ans: shs.100,000]**

   (c) A British traveller takes £500 (pounds) to Uganda. How many shillings can this be exchanged for if £1 = shs 3000 give your answer in words **[Ans: One million five hundred thousand shillings]**

2. (a) Find the amount of £ 85.20 in 8 years at $7\frac{1}{2}$% per annum simple interests. **[Ans: £136.32]**

   (b) What sum of money is invested for 5 years at $7\frac{1}{2}$% per annum yields shs. 150,000 simple interest. What is the amount at the end of this period? **[Ans: shs.550,000]**

   (c) Find how long it takes shs 96,000 invested at $8\frac{1}{3}$% simple interest to amount to sh 102,000. **[Ans: 9 months (0.75 years)]**

   (d) Find the interest earned when sh 450,000 is invested at 8% p.a. compound interest for 4 years. **[Ans: 162,220]**

3. In a certain country, the monthly gross income has certain allowances deducted from it before it is subjected to taxation. The allowances are:

married man	Shs 25,000
unmarried	shs 15000
transport	shs 3,000 per day
insurance	shs 15000
electricity	shs 18000
medical	shs 480,000 per anum

   Family allowance for 4 children: Shs 9000 for each child above 18 years, shs 12000 for each child below 18 but above 10 years and 15000 for each child below 10 years of age .Mukasa is a married man with 5 children 3 of them below 10, one aged 14 and the elder one 20 years old.

   (a) Find Mukasa's taxable income and the income tax he pays under the tax rates below given that he earns shs 960,000 monthly. **[Ans: taxable income = 715,000; Income tax = 90,475]**

Taxable income	Rate %
0 - 200000	2%
200001 - 400000	10%
400001 - 600000	18%
600001 - 800000	26.5%
above 800000	35%

(b) Express his income tax is a percentage of his gross monthly income. **[Ans: 9.2%]**

4. The table below shows the tax structure on taxable income of workers of National water and Sewerage co-operations.

Income (sh) per month	Tax rate
0 - 60,000	12%
60,001 - 150,000	17%
150,001 - 250,000	22.5%
250,001 - 350,000	34%
350,001 - 500,000	48%
above 500,000	50%

An employee of National water and Sewerage co-operations earns a gross salary of sh. 890,000 per month. The employee's allowances include the following:

Marriage allowance one tenth of his gross monthly income
Electricity allowance Sh. 35,000 per month
Insurance allowance shs 360,000 per annum
Transport allowance shs 25,000 per month
Medical allowance shs 20,000 per month
Housing allowance shs 60,000 per month

Family allowance for four children only. For children in the age bracket of 0 to 11 years shs 25,500 per child per month; between 11 and 18 years shs. 15,500 per child per month; and over 18 years shs 5, 500 per child per month.

(a) Calculate the employees' taxable income and the income tax paid, given that the employee is a married person with three children who are 5 years, 9 years and 13 years old. **[Ans: 564,500, 183,250]**

(b) What percentage of the employee's gross income goes to tax? **[Ans: 20.59%]**

5. Mr. Okello is a secondary school gateman whose monthly gross income is subjected to taxation after the following allowances have been deducted.

Marriage allowance $\frac{1}{20^{th}}$ of the gross income
Housing and electricity shs 480,000 per annum
Head of security department shs 60,000 per annum
Lunch and breakfast shs 15,000 per annum
Air time shs 60,000 per annum
Medical shs 240,000 per annum

If Mr. Okello earns a gross monthly income of shs 435,000

(a) Calculate the taxable income and income tax he pays under the tax structure below.

Income (shs)	Rate(%)
0 – 12,000	6.25
12001 – 45,000	9.0
45,001 – 87,000	13.5

87,001 – 145,000	18.0
145,001 – 217,000	26.0
Above 217,000	37.5

[Ans: 342,000 85,425]

(b) Determine the percentage of his gross monthly income paid in tax. **[Ans: 19.64%]**

6. (a) Kato bought a car at shs. 4,800,000 and sold it to Tom at shs. 3,600,000. Calculate the percentage loss. **[Ans: 25%]**

   (b) Otieno deposited some money in a bank that paid simple interest at 5% p.a, for a period of 5 years. How much money did he deposit if he withdrew a total amount of Kshs 15,000 from the bank? **[Ans: 12,000]**

7. A saleswoman earns a basic salary of shs 120,000 and a commission of 8% of the month's total sales. If the month's total sales were shs 1,350,000. find her income for that month. **[Ans: 228,000]**

8. (a) A forex bureau buys one US Dollar at Ugshs 1,900 and sells one Pound Sterling at Ugshs 3,450. Atim wants to exchange 3,000 US Dollars to Pound Sterling. How many pound Sterling will she get? **[Ans: £1652.17]**

   (b) The price of an article is shs 24,000. If a discount of 12% is given, find the selling price of the article. **[Ans: shs.21120]**

   (c) The selling price of an article is shs 6250. if the seller makes a profit of 25%, what is the cost price? **[Ans: 5000]**

9. A book was sold for shs.2160 at a loss of 10%. What did it cost? **[Ans: 2400]**

10. (a) A certain amount of money was invested at compound interest at a rate of 10% for 5 years. Given that at the end of the period the owner received sh. 500,000. Find the amount originally deposited. **[Ans: 310,460.66]**

    (b) The income tax rates of a certain country are as follows.

Income (shs)	Rates(%)
0 – 394,000	Tax free
394,001 – 694,000	30
694001 and above	36

(i) Find the income of an employee who paid shs. 385,200 of tax. **[Ans: 1,908,000]**

(ii) Express the income tax paid as a percentage of her income. **[Ans: 20.19%]**

11. In a certain country, income tax is levied as follows.

Taxable income (shs)	Rates(%)
0 – 10,000	10
10,001 – 20,000	25
20,001 - 30,000	30
30,001 - 40,000	45
Above 40,000	50

A person's monthly gross income has certain allowances deducted from it before it is subjected to taxation (This includes family relief and insurance).

The allowances are as follows:-

Married man	Sh. 12000
Unmarried man	Sh. 8500
Each child below 11 years	Sh.9000
Above 11 but below 18 years	Sh.7500
Insurance premium	Sh. 5000

(i) Sally earns sh 90,000. He is married with children aged 5,7, 12, 14 and 18 years. Given that he is insured and has claimed transport allowance of sh.1700, calculate the income tax he pays under the income tax rates above. **[Ans: 10,235]**

(ii) After two years, Sally's salary is increased but the allowances remained the same. If he is found to pay a tax amounting to sh 27,650, find his gross income and the Net income.
[Ans: Gross income = 125,000, Net income = 97,350]

12. (a) Nambuya deposited shs 270,000 in a bank that offers 12.5% compound interest per annum. After *n* years, her money had accumulated to shs 384,433.60. Calculate the value of *n*. **[Ans: 3 years]**

(b) Under hire-purchase, an item was bought by paying a deposit of shs 85,000 followed by six monthly installments of shs 120,000 each. If the hire-purchase price was higher than the cash price by 15%, calculate the cash price of the article. **[Ans: 700,000]**

13. (a) Pauline deposits shs. 200,000 in her Bank account at a simple interest rate of 10% per annum for 5years. Calculate the amount of money she received at the end of the period. **[Ans: 300,000]**

(b) (i) Tom bought a car at shs 4,500,000 and sold it at a profit of 7%. Calculate the selling price of the car. **[Ans: 4,815,000]**

(ii) A radio costs shs 300,000. Peter is given a 15% discount when he pays cash. How much does he pay for it? **[Ans: 225,000]**

(iii) Apio sells 125 shirts each at shs 9,000 and gets a 12% commission of the value of the shirts sold. Calculate her commission. **[Ans: 135,000]**

14. Below is an advertisement for a brand new laptop.

LAPTOP LAPTOP LAPTOP LAPTOP

HURRRY WHILE STOCK LASTS!

PRICE: SHS 1,200,000

CASH PAYMENT: GET A DISCOUNT OF 15%

HIRE PURCHASE: DEPOSIT 20% OF THE PRICE AND EITHER PAY SHS.390,000 MONTHLY FOR 3 MONTHS OR SHS.100,000WEEKLY FOR THIRTEEN WEEKS

(a) Calculate the cost of purchasing a laptop using;

(i) the weekly hire purchase terms. **[Ans: 1,540,000]**

(ii) the monthly hire purchase **[Ans: 1,410,000]**

(b) Find the savings one would make by paying cash rather than using weekly hire purchase terms.
[Ans: 520,000]

15. Income tax for all income earned is charged at rates shown below.

Income per annum	Rates(%)
0 - 1980	2
1981 - 3960	3
3961 - 5940	5
5941 - 7920	7
7921 - 9900	9
Over 9900	10

Mr. Mukasa earns a monthly gross income of shs 26,200. He also has a house allowance of shs 3200 per month and a family relief of shs 3500 per month.

(a) Calculate Mr. Mukasa's annual taxable income **[Ans: 234,000]**

(b) How much tax does he pay per month? **[Ans: 1,474.8]**

(c) What is his net salary per month? **[Ans: 24,725.2]**

16. A certain country's tax structure is such that a person's monthly income has certain allowances deducted from it before it is subjected to taxation. The allowances spelt out are as follows;
    - marriage allowances = $\frac{1}{10}$ of gross monthly income.
    - water and electricity = shs. 12,500 per month
    - housing allowance = shs. 40,000 per month
    - medical allowance = shs. 240,000 per annum

Taxable income (shs)	Rates(%)
00001 - 50,000	0
50,001 - 100,000	10
100,001 - 200,000	15
200,001 - 350,000	20
350,001 - 450,000	30
450,001 and above	35

    (a) Given that an employee is married and earns shs. 540,000 per month, calculate the tax she pays under the above income tax rates. **[Ans: 69,050/=]**
    (b) What percentage of her income is paid as taxi? **[Ans: 12.79%]**

17. A bank manger earns a gross salary of sh. 1,600,000 per month which includes an allowance of shs. 400,000 tax free. The rest of his income is subjected to an income tax which is calculated as follows;
    8.5% on the first sh. 500,000.
    14.5% on the next shs. 300,000
    30% on the next shs. 200,000
    45% on the next shs. 160,000
    65% on the remainder.
    Calculate the;
    (a) taxable income. **[Ans: 1,200,000]**
    (b) monthly tax paid by the bank manager. **[Ans: 244,000]**
    (c) percentage of the bank manager's monthly gross salary that goes to tax. **[Ans: 15.25%]**

18. A telecommunication company provides some benefits and allowances to its employees. The allowances given are:

Medical	-	shs. 80,000/= per month
Electricity	-	shs. 52,000/= per month
Marriage	-	shs. 70,000/= per month
Telephone bills	-	shs. 696,000/= per annum
Transport	-	shs. 1,500/= per day
Security	-	shs. 50,000/= per month
Insurance	-	shs. 1,800,000/= per annum
A child above 10 years	-	shs. 5,000/=
A child between 5 years and 10years	-	shs. 8,000/=
A child below 5 years	-	shs. 10,000/=

    The company gives family allowance for 2 children only.
    Drake is married with 4 children, one child is 13years old, another child is 8 years old and has twins of 3 years old. His monthly salary is shs. 1,180,000/=
    The country's tax structure is as shown below.

Tax income (shs)	Rates(%)
0 - 40,000	8
40,001 - 90,000	15
90,001 - 160,000	20
160,001 - 310,000	28
310,001 - 500,000	30
Above 500,000	40

Determine;

(a) the taxable income Drake paid in June 2010, with 30days. **[Ans: 655,000]**

(b) The income tax he paid as a percentage of his monthly salary. **[Ans: 15.74%]**

19. (a) Exchange rates in a Forex Bereau are that a US dollar to Uganda shillings is 1$ = 2785 and a pound sterling to US dollar is £1 = $ 2.59. How much pound sterling will a Ugandan trader get from shs. 4,570,185/=. **[Ans: £633.59]**

(b) The tax rates in a country for Government employees are as follows;

Taxable income (shs)	Rates(%)
0 - 80,000	2.0
80,001 - 190,000	5.0
190,001 - 280,000	7.5
280,001 - 380,000	12.0
380,001 - 490,000	15.0
Above 490,000	20.0

Every employee is entitled to the following allowances

Medical - 720,000/= per annum

Electricity - 40,000/= per month

Transport - 2,500/= per day

Housing - 90% of his monthly medical income

If the employee paid a monthly income tax of shs. 125,350/= in June, 2013. Calculate;

(i) the employee's taxable income **[Ans: 905,000]**

(ii) the employee's monthly gross income **[Ans: 1,134,000]**

20. An Askari at Mandela S.S in Hoima earns gross annual salary of shs 1,068,000 which includes the following allowances. Transport shs. 2,500 per month, water shs 1,050 per month, medical shs.60,000 per annum and children allowance of $\frac{1}{5}$ of his gross monthly earnings. The following tax strcture is applicable on the taxable income of his earnings per month.

Taxable income (shs)	Rates(%)
00001 - 5,000	9.0
5001 - 10,000	15.5
10,001 - 20,000	20.0
20,001 - 40,000	30.0
40,001 and above	45

Calculate;

(a) the taxable income for the askari

(b) the income tax paid by the askari

(c) his/her net income

(d) the tax paid as a percentage of the gross income earned.

21. (a) Tom bought a car at shs 3,800,000 and sold it to Patrick at a loss of 15%. How much did Patrick pay for it? **[Ans: 3,230,000]**

(b) Mr. Opio gets 7% commission on every fan he sells. In one week, he sells 4 large fans for shs 25,000 each and 9 small fans for shs 22,500 each. Calculate his total commission. [**Ans**: 21,175]

(c) A television set costs shs 300,000 cash. Ms. Apio buys it on hire purchase terms by paying a 15% deposit of the cash price and 12 equal monthly installments of shs 26,250 each. How much does she pay for it? **[Ans: 360,000]**

22. A private school's income tax structure is such that a teacher's monthly income for the month of October has certain allowances deducted from it before its subjected to taxation. The allowances are spelt out as follows:

Marriage allowance is one-tenth of the gross monthly income. Family relief is shs. 240,000 per annum. Housing shs. 35,000 per month. Medical shs. 120,000 per annum. Transport is shs 1000 per day. Departmental allowance shs. 15,000 per month. Administrators allowance shs. 50,000 per month. Family allowance for 3 children only at the following rates.

Class level	Allowances (shs)
A - level	6,800
O- level	7,200
Primary	9,000

Mr. Opio is married with 4 children and 2 of them are in primary, one in O-level and the other at A-level and he is not a head of department. Given that he earns a gross monthly income of shs. 680,000 calculate:

(a) The taxable income and the income tax he pays under the tax rates shown in the table below.
[Ans: taxable income = 485,000, income tax = 156,840]

Taxable income (shs)	Percentage tax rate
0 - 15,000	8
15,001 - 84,000	16
84,001 - 170,000	25
170,001 - 285,000	34
285,001 - 435,000	40
Above 435,000	48

(b). Determine the percentage of his gross monthly income paid in tax. **[Ans: 23.06%]**

23. Income tax in a certain country is calculated after deducting the following allowances. Marriage Sh 91,000 which is one – tenth of the gross monthly income, transport ; Sh 60,000 per month, insurance; Sh 372,000 per annum, medical;sh28,000 per month, housing; Sh 42,000, electricity ;Sh 15, 000 and water ;Sh 500 per day. Family allowance for only 3 children at the following rates; Sh21, 500 for each child below 10 years,Sh 20, 000 for each child between10 years and 15 years;Sh 15,000 for each child above 15 years. The tax rates are as shown below

Taxable income (shs)	Rates(%)
0 - 80,000	10
80,001 - 180,000	12
180,001 - 280,000	15
280,001 - 380,000	17.5
380,001 - 500,000	18
Above 500,000	20

Mr. Okiror is married and has 4 children aged 7, 9, 14 and 20 years.

(a) Calculate Mr.Okiror's taxable income. **[Ans: shs 565,000]**

(b) What percentage of Mr.Okiror's gross income goes to tax? **[Ans: 9.57%]**

24. Juma deposited shs. 1,600,000 in a bank which paid compound interest at the rate of 12% per annum. At the end of 5 years, he withdrew all his money. Determine how much money he withdrew.
[Ans: 2,819,746.693]

25. (a) Peter deposited shs.2,500,000 in a bank which offers a compound interest of 15% per annum. How much money did he have in the bank at the end of two years? **[Ans: 3,306,250]**

(b) The cash price of a radio is shs 720,000. It can be bought on hire purchase terms by making a deposit of 30% of the cash price and then paying 8 monthly instalments of shs 85,000 each.

(i) Find the cost of the radio on hire purchase terms **[Ans: 896,000]**

(ii).How much more does one pay on hire purchase rather than on cash terms? **[Ans: 176,000]**

26. The tax structure in a certain country on the taxable income is as follows:

Income(shs) per month	Tax rates (%)
01 - 130,000	Tax free
130,001 - 280,000	9.0
280,001 - 480,000	12.5
480,001 - 880,000	20.0
880,001 and more	30.0

Mr. Mugalu's gross income is shs978,000. The allowances include;

Allowances	**Amount (shs)**
Transport	75,000 per months
Family	A tenth of gross pay
Medical	150,000 per annum
Insurance	15,000 per month
Housing	80,000 per months

Determine Mr.Mugalu's;

(a) Total monthly allowances **[Ans: 280,300]** (b) Taxable income **[Ans: 697,700]**

(c) Tax paid annually **[Ans: 984,480]** (d) Net pay. **[Ans: 895,960]**

27. A vendor receives commission of 5% on the first total sales up to Shs 100,000 and 8% on the total sales exceeding Shs100,000. Determine the total sales of the vendor who received Shs 16,200 commission.
[Ans: 240,000]

28. A worker is entitled to the following allowances;

Insurance Shs.1,524,000 per annum, Housing Shs.120,000 per months, Fuel of 5 litres per day, Electricity Shs.60,000 per month, Medical Shs.972,000 per year., His income is further subject to the tax structure below;

Taxable income (shs)	Rates(%)
01 - 150,000	8
150,0001 - 330,000	10
330,001 - 530,000	12
530,001 - 750,000	15
750,001 - 1,030,000	20
1,030,001 and above	25

N.B: 1 month = 30 days.

If the worker paid an income tax of shs.173,000 and the price per litre of fuel is shs.3,800. Calculate his;

(a) taxable income **[Ans: 1,150,000]** (b) monthly net income **[Ans: 1,935,000]**

29. The tax structure of a certain country is as below.

Taxable income (shs)	Rates(%)
01 - 100,000	Tax free
100,001 - 200,000	12.5
200,001 - 300,000	18
300,001 - 400,000	25
400,001 - 500,000	28
500,001 - 600,000	30
Above 600,0000	45

The following allowances are offered to employees: transport **shs50,000** per month, housing/rent **shs192,000** per annum, marriage **shs24,000** per month, insurance **shs720,000** per annum. Water shs**1000** per day, electricity **shs40,000** and every child 20 years and above **shs3500**.

Mr.Ssenyange is married with five children aged 3,7,11,18 and 24 years. He earns a gross monthly income of shs800,000. Calculate

(a) Total monthly allowances **[Ans: 201,900]** (b) Monyhly taxable income **[Ans: 598,100]**

(c) Income tax as a percentage of his gross income **[Ans: 14.12%]** (d) Net monthly income **[Ans: 687,070/=]**

30. Below is an advertisement for a brand new dinning set.

BRAND NEW DINNING SET BRAND NEW DINNING SET

HURRY WHILE STOCK LASTS!

PRICE: SHS 2MILLION

CASH: DISCOUNT OF 5%

HIRE PURCHSE: DEPOSIT 30% OF THE PRICE AND PAY EITHER SHS 120,000 MONTHLY FOR 13 MONTH OR SHS 50,000 WEEKLY FOR 30 WEEKS.

(a) Calculate the amount one would save by paying cash rather than using the monthly hire purchase term. **[Ans: 260,000]**

(b) If the whole sale price of the dinning set was 10% below the price. Calculate the percentage profit the seller gets on the weekly hire purchase terms. **[Ans: 16.67%]**

31. Babirye is a sales lady in a biscuit company. She receives a salary of shs4,000 per month and a commission of 10% of the total sales for the first 1000 biscuits sold and further discount of 12% of the sales for biscuits in excess of 1,000. The biscuits are sold at shs2,000 each. For June she sold 1500 biscuits.

(i) How much did she receive at the end of the months? **[Ans: 324,000]**

(ii) For July the biscuits sold increased by 10%. How much did she receive at the end of July? **[Ans: 360,000]**

(iii) For a months of August she received shs420,000. How many biscuits did Babirye sell during the month of August? **[Ans: 1900 biscuits]**

32. (a) Mr. Zziwa borrowed shs9,000,000 for 3 years at a compound interest of 5% per annum. How much money does he repay by the end of 3 years? **[Ans: 10,418,625]**

(b) John invested Shs 3 million at a 25% simple interest rate per annum. How much money does she receive by the end of five years? **[Ans: 6,750,000]**

(c) Mr. Lubuulwa bought 10 sacks of sugar at shs50,000each, 10 bags of beans at shs30,000 each, 10 sacks of rice at shs100,000 each and 10 sacks of groundnuts at shs120,000 each. These items were offered 5%, 10%, 6% and 4% discounts respectively.

(i) how much money did he spend? **[Ans: 2,837,00]**

(ii) what was his total discount? **[Ans: 163,000]**

33. (a) Okello bought a car at 30 millions. If his car depreciates at a rate of 25% p.a, determine the amount he will sell his car after 4 years. **[Ans: 9,492,187.5/=]**

(b) a car costs 9600 dollars and depreciates at a rate of 15% per annum. What is the value of the car after 3 years? **[Ans: 5,895.6 dollars]**

34. The value of a certain excavator increased by 20% in the first year, depreciated by 10% in the second year and again increased by 10% in the third year. If the valueof the excavator at the end of the third year is 297,000,000 million, find it's original value. **[Ans: 250,000,000]**

35. Below is the advertisement for a Refregerator.

REFREGERATOR FOR SALE!

TERMS: CASH shs800,000 LESS 5% DISCOUNT

OR

FOR MONTHLY HIRE PURCHASE: DEPOSIT 300,000 AND THEN 4 MONTHLY INSTALLMENTS OF shs150,000 EACH OR 8 WEEKLY INSTALLMENTS OF 90,000 EACH

Calculate

(a) The cash price **[Ans: 760,000]** (b) The monthly hire purchase **[Ans: 900,000]**

(c) Weekly hire purchase **[Ans: 1,020,000]**

(d) The savings a customer would make by buying a radio on cash basis rather than

(i) Weekly hire purchase. **[Ans: 260,000]** (ii) Monthly hire purchase. **[Ans: 140,000]**

36. In a certain country, the monthly gross income has certain allowances deducted from it before it is subjected to taxation. The allowances are as follows;

Allowance	**Amount (shs)**
Marriage	18,000
single	12,000
Each child below 11 yrs	5,000
Each child above 11 yrs and below 18 yrs	7,000
Insurance premium	12,000

The income tax rates are as indicated in the table below.

Taxable income	Rates(%)
0 - 50,000	10
50,001 - 100,000	15
100,001 - 200,000	20
200,001 - 300,000	25
300,001 - 400,000	30
400,001 - 500,000	35
500,001 - above	40

Thomas is married and a father of John, Mary and Fred of ages 9, 10 and 13 years respectively. Given that he is insured and pays 654,000 as income tax per month, calculate;

(i) His gross monthly income **[Ans: 1,875,750]**

(ii) His taxable income **[Ans: 1,828,750]**

(iii) The percentage of his monthly income he pays as income tax. **[Ans: 34.87%]**

37. A vehicle has a market price of shs12 million. A customer can buy either by cash and get a discount of 15% or by a 55% depofosit and 12 monthly installments 550,000. Find how much he would pay by hire purchase than paying cash. **[Ans: 600,000]**

38. James is employed with an electricity installation company that pays him a gross salary of shs6,600,000 per year. He is married with 4 children, two of whom are aged 17 years and 19 years respectively while the other two are aged 11 years and 14 years respectively. The company pays allowance for only 3 children for every employee. A summary of allowance is as follows: Marriage; $\frac{1}{10}$ of gross monthly income, Medical ; 300,000 per year, Transport; 2000 per day, Children above 15years but below 29 years of age; 1000 per month, Child above 10 years but below 15 years of age; 6000 per months.

(a) Calculate James's taxable income for the month of june **[Ans: 397,000]**

(b) (i) calculate the income tax John pays in June using the tax rate below; **[Ans: 75,750]**

Income (shs) per month	Tax rate
0 - 100,000	10
100,001 - 150,000	15
150,000 - 220,000	20
220,001 and above	25

(ii) Calculate the income tax he pays as a percentage of his taxable income. **[Ans: 13.77%]**

39. In a certain organization the income tax system operates as follows;

Rates on taxable income:

30% on the first $4000

40% on the next $2000

50% on the next $2000

60% on the next $2000

65% on any amount in excess of the above

Allowances:

$\frac{2}{9}$ of earned income is tax free

Single $400, married $750

Children; one child($200), 2 children ($450), 3 children ($700)

Insurances and other expenses.

A doctor is married with two children, has an annual salary of $21600. He is entitled to $300 for life insurance premium.

Calculate;

(i) How much income tax would he pay? **[Ans: £7645]**

(ii) What percentage of his annual salary he would pay as income tax? **[Ans: 35.39%]**

THE SET THEORY

RECALL

Set notation

◊ These provide a description of symbols used in set language.

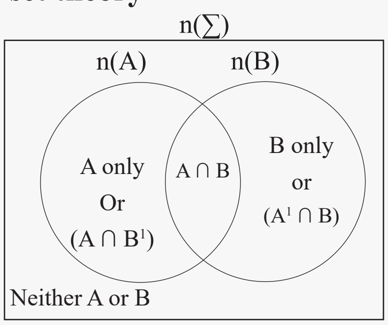
◊ The language in set should be mastered and interpreted well.

The set theory may be categorised into two;

(a) Two set theory. (Two sets are involved)

(b) Three set theory (Three sets are involved)

Two set theory



∑ → Universal set (includes members of the union set and those outside it.

'∩' → Intersection of sets. (This is a set consisting of all common members in the given sets.

n(A) → number of elements in set A

n(B) → Number of elements in set B

A ∪ B → all elements which belong to either A or B or both A and B without repetition.

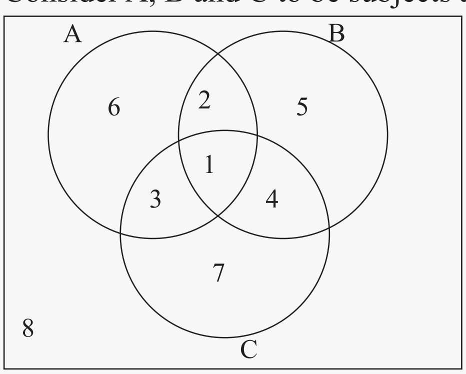
$A \cup B = A \cap B + A \cap B^1 + A^1 \cap B$ $\quad A \cup B = A \text{ only} + B \text{ only} + A \cap B$

A ∩ B means elements in A but are also in B

(b) Three set theory

This involves large number of elements and it involves several methods of solving unknowns where need be.

Consider A, B and C to be subjects attended by students.



The regions 1, 2, 3 ...... 8 are given by the following sets.

(i) $A \cap B \cap C = 1$ (intersection of all sets)

(ii) $A \cap B \cap C^1 = 2$ $(A \cap B)$ only

(iii) $A \cap B^1 \cap C$ or $(A \cap C)$ only $= 3$

(iv) $A^1 \cap B \cap C$ $(B \cap C)$ only $= 4$

(v) $A^1 \cap B \cap C^1$ (B) only $= 5$

(vi) $A \cap B^1 \cap C^1$ (A) only $= 6$

(vii) $A^1 \cap B^1 \cap C$ (C only) $= 7$

(viii) $(A \cup B \cup C)^1 = 8$

(ix) A and B or $A \cap B = (A \cap B)$ only + $(A \cap B \cap C$

$= 2 + 1 = 3$

(x) A and C or $A \cap C = (A \cap C)$ only + $(A \cap B \cap C$

$= 3 + 1 = 4$

(xi) B and C or $B \cap C = (B \cap C)$ only + $(A \cap B \cap C$

$= 2 + 1 = 3$

Note; atmost two subjects from the language , we consider 2 and below including these who do not offer any subject. $2 + 3 + 4 + 6 + 5 + 7 + 8 = 35$ (atmost two subjects but $2 + 3 + 4 + 1 = 10$ (students who offer atleast two subjects). For only one subject, consider 6, 7 and 5 and add all of them together.

N.B: Consider the language used in the sets and solve the challenges under this topic.

LANGUAGE

◊ "OR" → means Union, ie A or B $A \cup B$

◊ "And" → means intersection of sets i.e A and B means A n B.

◊ "Neither - nor" - neither A nor B means members who do not belong to the union of the sets in context.

◊ Compliment of a set: Is a set consisting of all elements in the universal set that are not in the given set. e.g A^1, means members who do not belong to A. But it should be noted that members of A^1 also include members outside the union set.

◊ Atmost: Means that number and below with zero inclusive (members outside the union inclusive)

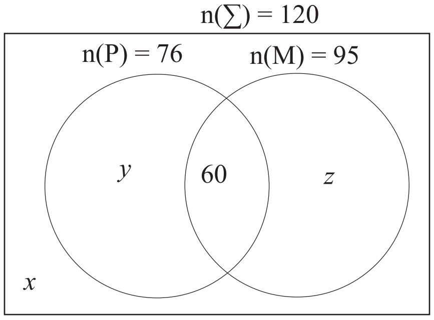
◊ Atleast: means that number and above with the intersection inclusive.

◊ More than: Above that member in context. i.e Find the number of students doing more than one subject. Therefore look for students who do two and above subjects.

TRIAL QUESTIONS

1. In S4, there 30students, 18 are doing math(M), 15 are doing physics(P) and 13 chemistry(C). The number of students who are doing all the three subjects equals the number of those students who are not doing any of these subjects. Ten Students do M and C, 8 do M and P and 3 do only C and P. Determine;
   (i) the number of students doing all the three subjects **[Ans: 5]**
   (ii) the number of students who are doing only one subject **[Ans: 9]**
   (iii) the probability that a student selected at random from the class does at least two of these subjects. **[Ans $\frac{8}{15}$]**
2. The venn diagram shows girls who offer physics (P) and Maths (M).

$n(\sum) = 120$



Find the;

(i) Value of x, y and z. **[Ans: y = 16, z = 35, x = 9]**

(ii) $n(P \cup M)$ **[Ans: $n(P \cup M) = 111$]**

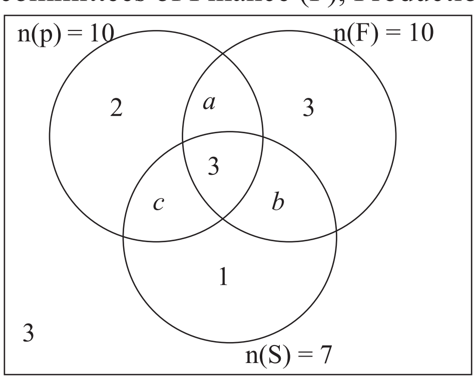
3. In a group of 16 peoples, 7 ate Matooke (M), 8 ate Rice (R) and 3 ate neither Matooke nor Rice. Represent the information on a Venn diagram and hence find $n(M \cap R)$ **[Ans: $n(M \cap R) = 2$ people]**
4. Given that M = { All prime numbers less than 20} and N = {All triangle numbers less than 20}, find $n(M \cap N)$ **[Ans: $n(M \cap N) = 1$]**
5. If set T = {All triangle numbers less than 20}, and F = {All factors of 12} Find the members of $F \cap T$. Hence find $n(F \cap T)$. **[Ans: $F \cap T = \{1, 3, 6\}$, $n(F \cap T) = 3$ members]**
6. Given that { all odd numbers less than 20} and { all multiples of three less than 20}, find $n(D \cap M)$ **[Ans: 3 members]**
7. Given that F = { All factors of 24} and G = {all factors of 30}. Find $n(F^1 \cap G)$ where F^1 is the compliment of F. **[Ans: 4 members]**
8. In a class of 80 pupils, 45 like English(E) and 50 like Maths(M).
   (i) Represent this information on a venn diagram
   (ii) Find the number of pupils who like both E and M. **[Ans: 15 pupils]**

9. Given that n(A) = 7 , n(B) = 9 , $n(A \cup B)^1 = 2$ and n($\sum$) = 15. Use a venn diagram to find;
   (i) $n(A \cap B)$ **[Ans: 3]** (ii) $n(A \cap B^1)$ **[Ans: 4]**

10. In a class of 70girls, 30like Math(M), 40 like Physics(P) and 20 like English(E). If 10 like both Math and Physics, 8 like English and Physics, 9 like both Math and English and 4 don't like any of the three subjects.
    (a) Represent the information on a venn diagram
    (b) Use the venn diagram to find the number of girls that;
        (i) like all the three subjects **[Ans: 3 girls]**
        (ii) don't like Math **[Ans: 40 girls]**
    (c) If a student is picked at random, what is the probability that she likes only one subject. **[Ans: $\frac{9}{14}$]**

11. In a certain school there are 50 students who visited three towns; Kampala(K), Mbarara(M) and Jinja(J). 24 visited Kampala, 26 visited Mbarara and 29 visited Jinja. 9 visited both Kampala and Jinja, 13 visited Mbarara and Jinja. Each of these students visited at least one of the three towns.
    (a) Represent the above information on a venn diagram.
    (b) Find the number of students who visited
        (i) all the three towns
        (ii) at most two games.
    (c) Find the probability that a student picked at random, visited either Kampala or Mbarara but not Jinja.

12. The venn diagram below shows the representation of members of a community council to three different committees of Finance (F), Production(P) and Security(S).



    (a) Determine the values of a, b and c.
    (b) Find the;
        (i) total number of members that make up the community council
        (ii) number of members who belong to at least two committees.
    (c) Given that a member is chosen at random from the council, what is the probability that the member belongs to;
        (i) only one committees?
        (ii) atmost two committees?

[Ans: (a) $a = 3, b = 1, c = 2$ (b)(i) 18, (ii) 9, (c)(i) $\frac{1}{3}$, (ii) $\frac{5}{6}$]

13. In S.4 there are 50 students who offer three subjects namely Luganda(L), Computer (C) and Agriculture(A). 24 offer Agriculture, 29 offer computer and the number offering Luganda is 3 less than the number offering Computer. 9 offer both Agriculture and Computer, 13 offer both Luganda and Computer, 11 offer Agriculture and Luganda. Each of these students offer at least one of the three subjects.
    (a) Represent the above information on a Venn diagram
    (b) Find the number of students that offer
        (i) all the three subjects **[Ans: 4 students]**
        (ii) only one subject **[Ans: 25 students]**
    (c) Find the probability that a student selected at random offers
        (i) Luganda only **[Ans: $\frac{3}{25}$]**
        (ii) two of the subjects **[Ans: $\frac{21}{50}$]**

14. In a certain school there are 50 students who play three games, namely; Chess, Tennis and Volleyball. 24 play Chess, 26 play Tennis and 29 play Volleyball, 9 play both Chess and Volleyball, while 13 play both Tennis and Volleyball, 11 play both Chess and Tennis. Each of these students plays atleast one of the three games.

(a) Represent the above information on a venn diagram.

(b) Find;

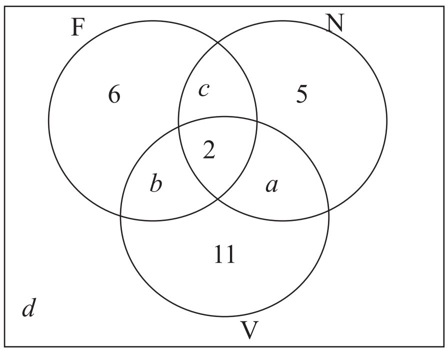
(i) how many students play all the three games **[Ans: 4 students]**

(ii) the number of students who play atmost two games **[Ans: 46 students]**

(iii)the number of students who play atleast two games **[Ans: 25 students]**

(iv) the probability that a student selected at random does not play Tennis **[Ans: $\frac{12}{25}$]**

15. The diagram below shows 35 students who participated in Football (F), Netball (N) and Volleyball (V) tournament.



If n(V) = 20, n(N) = n(F) = 12,

(a) Find the values of *a*, *b*, *c* and *d* **[Ans: a = 4, b = 3, c = 1, d = 3]**

(b) Determine; (i) $n(F \cap N \cap V^1)$ **[Ans: 1]**

(ii) $n(F^1 \cap V \cap N^1)$ **[Ans: 11]**

(c) Find the probability that a student picked at random plays neither game. **[Ans: $\frac{3}{35}$]**

16. In a certain town, 92 tourists were found to have visited other countries. 28 had visited Zambia(Z) , 36 had visited Kenya(K) and 50 had visited Tanzania(T); 8 had visited Zambia and Kenya, 14 had visited Kenya and Tanzania. The number of those who had visited all the three countries was equal to those that had visited Zambia and Tanzania only. 6 people did not visit any of the three countries.

(a) Represent this information in a venn diagram.

(b) Find the number of tourists who had visited

(i) all the three countries **[Ans: 6]**

(ii) atleast two countries **[Ans: 22]**

(iii) Zambia and Tanzania **[Ans: 12]**

17. In a certain school, there are 87 students in S.3. Of these, 43 play hockey (H), 42 play football (F) and 47 play Volleyball (V). 15 play hockey and volleyball, 17 play volleyball and football and 21 play hockey and football. Each student plays atleast one of the three games.

(a) Using a venn diagram, represent the information given above.

(b) Determine the number of students who play all the three games **[Ans: 8 students]**

(c) How many students play only two of these games. **[Ans: 29 students]**

(d) If a student is selected at random from the class, find the probability that the student plays only one of the games. **[Ans: $\frac{50}{87}$]**

18. In Ridar hotel, thirty nine guests ordered either Matooke, Cassava or rice. 24 ordered for Matooke(M), 16 ordered for Rice(R) and 17 ordered for Cassava(C). Those who ordered for Matooke and Cassava were more than those that ordered for both Cassava and Rice by one person. 9 ordered for Matooke and rice. 2 ordered for all the three.

(a) Represent the information on a venn diagram.

(b) Find how many guests that ordered for both cassava and Rice. **[Ans: 5]**

(c) If a guest is picked at random from the hotel, what is the probability that a guest did not order for Matooke? **[Ans: $\frac{5}{13}$]**

19. In a training college, students must take atleast one of the following subjects. Fine Art(A), Physical Education(P) and Music (M). Of the 70 stduents in the first year, none takes P and M. 26 take P only. Of the 35 students taking A, 20 have specialized in this subject alone. The number of students taking P and A is three more than the number taking M and A.

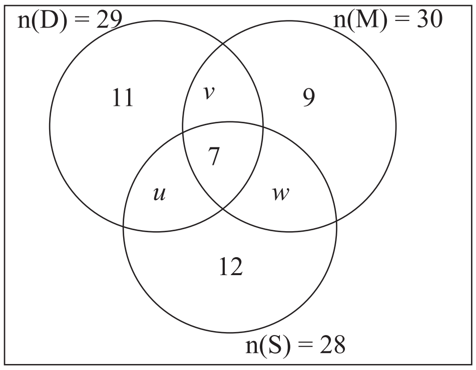
Using a Venn diagram, find the number of students taking

(i) A and M **[Ans: 6 students]**

(ii) M only **[Ans: 9 students]**

(iii) Two subjects only **[Ans: 15 students]**

(iv) Only one subject **[Ans: 55 students]**

20. 

The diagram above shows the representation of students of a S. 4 class who belong to the clubs of debating (D) Mathematics (M) or Science (S) which are in their school.

(a) Determine the values of u, v and w. **[Ans: $u = 3$, $v = 8$, $w = 6$]**

(b) Find the total number of students in the S.4 class. **[Ans: 56 students]**

(c) If a student is selected at random from the class, find the probability that the student belongs to atmost two clubs? **[Ans: $\frac{7}{8}$]**

21. At noon on Tuesday, a group of news paper vendors were asked whether they still had News papers from the Monitor(M), the New Vision(N) and the Red paper(R). All the vendors had atleast one of the three papers. 28 had the monitor, 29 had the New Vision and 30 had the Red Paper. 15 had the Monitor and the New Vision, 9 had New Vision and Red Paper, 8 had the Monitor and the Red Paper while 8 had Monitor only.

(a) Represent the information above on a Venn diagram.

(b) Find the number of Vendors with;

(i) all the three types of papers. **[Ans: 3 vendors]**

(ii) only two types of papers left **[Ans: 23 vendors]**

(iii) atleast two types of papers **[Ans: 26 vendors]**

(c) Find the probability that a vendor picked at random from the group still had the Monitor and Red Paper News papers only. **[Ans: $\frac{5}{58}$]**

22. In a certain class, 20 students like German, 10 like both French and Germany, 9 like both French and Kiswahili, 7 like Kiswahili and Germany only and 2 like none of these languages. 22 like atleast two of the languages, 12 like one language only and 11 like either Kiswahili or German or both but not French.

(a) Repeat this information on a suitable Venn Diagram

(b) Find the number of students who like

(i) all the three languages **[Ans: 4 students]**

(ii) Kiswahili only **[Ans: 1 student]**

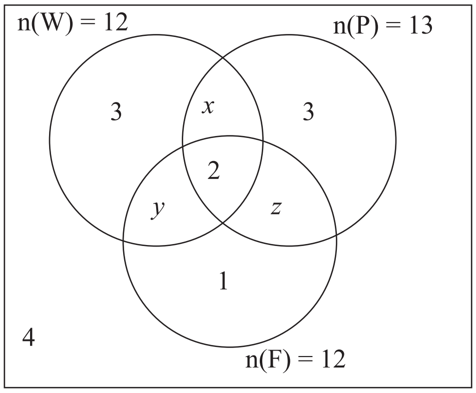
(iii) French only. **[Ans: 8 student]**

(c) find the number of students in the class. **[Ans: 36 students]**

(d) What is the probability that a student selected at random likes neither Kiswahili nor Germany?

[Ans: $\frac{5}{18}$]

23. The venn diagram below shows the members of a district council who sit on three different committees of works (W), production (P) and Finance (F) .



(a) Determine the values of x, y and z **[Ans: $x = 3, y = 4, z = 5$]**

(b) Find the total number of members who;

(i) make up the district council. **[Ans: 25 members]**

(ii) belong to more than one committee. **[Ans: 14 members]**

(c) Given that a member is selected at random from the council, find the probability that the member belongs to only two committees. **[Ans: $\frac{12}{25}$]**

24. In a class of 75 students, 38 students like Matooke (M), 44 students like Posho(P) while 33 students like Rice (R). Of these students 17 like both Matooke and Posho, 16 like both Matooke and Rice while 20 like both Posho and Rice. Only 6 of the students do not like any of the foods.

(a) Represent the above information on a Venn diagram.

(b) Determine the number of students who like all the three foods. **[Ans: 3 students]**

(c) How many students like only two of these foods? **[Ans: 44 student]**

(d) If a student is selected at random from the class, find the probability that the student likes only one of the foods. **[Ans: $\frac{6}{25}$]**

25. In a class of 39 students, 24 offer Divinity(D), 16 offer History (H) and 17 offer Commerce (C). Those who offer Divinity and Commerce are more than those who offer Commerce and History by one student. 9 offer both Divinity and History. 2 offer all the three.

(a) Represent the information on a Venn diagram.

(b) Find how many students offer both Commerce and History. **[Ans: 5]**

(c) If a student is picked at random from the class, what is the probability that the student does not offer Divinity? **[Ans: $\frac{5}{13}$]**

26. There are 100 students in A– Level taking various subject combinations. Of these, 60 students take Maths(M) , 45 take Physics(P) and 40 take Chemistry (C). Out of these, 60 students taking M, 16 do neither P nor C. of the 45 taking P, 8 do neither M nor C. Of those taking C, 5 take neither M nor P. 7 take both M and C but not P.

(a) Find the number who take

(i) M and P **[Ans: 37]**

(ii) P and C but not M **[Ans: 0(zero)]**

(iii) all the three subjects **[Ans: 28]**

(iv) none of these subjects **[Ans: 27]**

(b) Find the probability that a student picked at random offers only one subject **[Ans: $\frac{29}{100}$]**

27. In an examination, 93 candidates attempted at least Qn1, Qn2 and Qn3. Qn 1 was attempted by 67 candidates, Qn2 by 46 candidates and Qn3 by 40 candidates , 28 attempted Qn1 and Qn2, 26 attempted Qn1 and Qn3 and those who attempted all the three questions were four less than those who attempted Qn2 and Qn3 only.

(a) Represent the above information on a venn diagram.

(b) How many candidates attempted Qn2 and Qn3 but not Qn1 **[Ans: 6]**

(c) If a candidate is picked at random form the examination, what is the probability that the candidate attempted atleast two questions. **[Ans: $\frac{58}{93}$]**

28. In watching Premier football team games, 56 members watched Arsenal (A), 48 watched Liverpool (L) and 36 watched Chelsea (C). All the members watched one or more teams except 6 members who did not have entry fee. 32 watched Arsenal only, 10 watched Liverpool only, 8 watched Liverpool and Arsenal but not Chelsea. 39 watched more than one team.

    Use a Venn diagram to find how many members watched;

    (a) the teams **[Ans: 86 members]**

    (b) Chelsea only **[Ans: 5 members]**

    (c) more than two teams. **[Ans: 15 members]**

29. On a wedding ceremony 71 guests were asked which flavours of Mirinda (M), Novida (N) and Fanta (F) they each prefer. It was found out that an equal number of guests preferred M and N. 10 guests preferred M and F, 11 guests prefer F and N while 6 preferred M and N only. 26 preferred F and 5 preferred M only. The number of guests who preferred F only doubles those who preferred N only.

    (a) Represent the above information on a venn diagram

    (b) Find the number of guests who; (i) Preferred N only **[Ans: 4 guests]**

    (ii) Preferred all the flavours **[Ans: 3]**

    (iii) did not like any of the three **[Ans: 30]**

    (c) If a guest is chosen at random from the group, find the probability that he/she preferred at most two drinks. **[Ans: $\frac{68}{71}$]**

30. In Luwero High School 32 students were asked about their favourite games and the responses were as follows:

    17 play football (F)

    20 play hockey (H)

    18 play volleyball (V)

    11 play Fand H

    2 play V and H but not F.

    5 play F and V but not H.

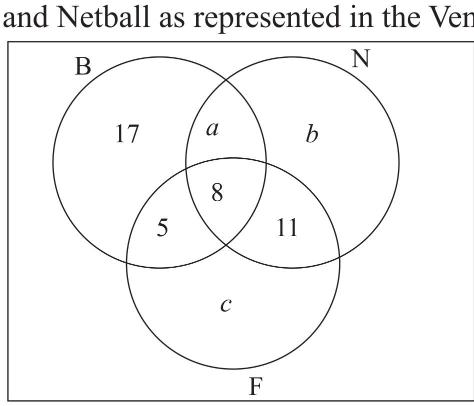
    Determine :

    (a) the number of students who play all the 3 games. **[Ans: 5 students]**

    (b) the number of students who play only one game. **[Ans: 14 students]**

    (c) what percentage of the students play at least 2 games **[Ans: 56.3%]**

31. In a class nof 94 S.3 students, each student is to play at least one of the three games; Badminton, Football and Netball as represented in the Venn diagram below.



If n(B) = 40 and n(N) = 55. Find the values of a, b and c.

(a) What percentage of students play football?

(b) Find the probability that a student selected at random plays at most 2 games.

[Ans: (a) 43.6% (b) $\frac{43}{47}$]

32. A school receives three National Newspapers everyday, namely The New Vision (V), The Monitor (M) and Red pepper (R). In the same school there is an 'A' level class of S.6 science consisting of 90 students. Of these students 13 read both V and M, 18 read both V and R, 16 read both M and R. 40 read V, 37 read M and 46 read R. The number of students in that same class who read all the three newspapers is equal to the number who do not read any of them.

(a) Represent the above information on a Venn diagram

(b) Determine the number of students who read all the three newspapers. **[Ans: 7 students]**

(c) How many students read only two of the newspapers? **[Ans: 26 students]**

(d) If a student is selected at random from the class, find the probability that the student reads only one of the newspapers. **[Ans: $\frac{5}{9}$]**

33. In a certain school there are students who play football (F), Tennis (T) or volleyball (V). 24 play football, 25 play Tennis and 29 play volley ball. 11 play both F and T, 10 play both T and V while 13 play both F and V. The number of students who play tennis or volley ball but not football is equal to twice those who play neither of the three games. If those who play neither of the games are 12,

(a) Represent the above information on a venn diagram.

(b) Find the;

(i) total number of students in the school. **[Ans: 60 students]**

(ii) number of students who play only two games. **[Ans: 22 students]**

(c) Find the probability that a student chosen at random plays not more than one game. **[Ans: $\frac{17}{30}$]**

34. A school has a teaching staff of 22 teachers 8 of them teach mathematics, 7 teach physics and 4 teach chemistry. Three teach both mathematics and physics and one teaches mathematics and chemistry. No teacher teaches all the three subjects. The number of teachers who teach physics and chemistry is equal to that of those who teach chemistry but not physics

(a) Represent the above information on a venn diagram

(b) Find the number of teachers who teach

(i) mathematics only **[Ans: 4 teachers]**

(ii) none of the three subjects **[Ans: 9 teachers]**

35. Seventy five candidates sat for a paper containing 3 sections A, B and C. Forty-five attempted section C, 26 attempted section A but not C, 10 attempted all the three sections. Twenty-eight attempted section A and C, 23 section B and C and 4 could not attempt any of the sections. If 20 attempted section A and B:

(a) Represent this information in a Venn Diagram.

(b) Determine how many candidates attempted (i) section B **[Ans: 33 candidates]**

(ii) only one section. **[Ans: 20 candidates]**

(c) A candidate was selected at random, what is the probability that he/she attempted at least 2 sections. **[Ans: $\frac{17}{25}$]**

36. A tourist office looks at 20 East African national parks to see which of them have elephants(E), lions(L) and rhinoceros(R). They find that; $n(E \cap L^1 \cap R^1) = 2$, $n(L \cap E^1 \cap R^1) = 1$, $n(R \cap L^1 \cap E^1) = 0$, $n(E \cup L \cup R)^1 = 3$, $n(E \cap L \cap R^1) = 8$.5 parks have elephants and rhinoceros and 4 parks have lions and rhinoceros.

(a) Draw a venn diagram to show this information

(b) How many parks had all three of these animals? **[Ans: 3 parks]**

(c) What is the probability that a park chosen at random has atleast two of these animals? **[Ans: $\frac{7}{10}$]**

37. In a mathematics class the teacher told students to bring a pen (P), a graph book (G) and a ruler (R) for use. During the next lesson it was found out that only 16 students brought all the items. 5 students did not have any of the items. 13 did not have a pen,14 students did not have a graph book and 20 did not have a ruler. One student only had a pen, 2 students had only a graph book and no student had only a ruler.

(a) Represent the above information on a venn diagram.

(b) How many students;

(i) were in the class? **[Ans: 50]**

(ii) had a pen and a ruler only? **[Ans: 8]**

(c) If a student is selected from this class at random, find the probability that he had;

(i) at least 2 items. **[Ans: $\frac{21}{25}$]**

(ii) only one item. **[Ans: $\frac{3}{50}$]**

38. A group of students play either football (F) Volleyball (V) or Tennis(T), 21 play F, 17 play V and 18 play T. 7 play F and V, 11 play T and F while 3 play only T and V. Those playing V only exceeds those playing the three games by 2.

(a) Draw a venn diagram representing the given information.

(b) How many students play Tennis only? [**Ans: 4]**

(c) How many students are in a group? **[Ans: 35]**

(d) Find the probability of picking on a student playing atleast two games. **[Ans: $\frac{16}{35}$]**

39. In a random sample of 1000 men, the following facts emerged. 271 of the men were bald (B), 248 of the men wore spectacles (S), 251 of the men had false teeth (T), 64 men were both bald and wore spectacles, 97 men were both bald and had false teeth, 59 wore spectacles and had false teeth. 434 were not bald, did not wear spectacles and had all their own teeth.

(a) Represent the information on a venn diagram.

(b) How manywere bald, wore spectaces and had false teeth. **[Ans: 16]**

(c) How many men were bald but neither wore spectacles nor have false teeth? **[Ans: 126]**

(d) If a man is chosen at random, what is the probability that he wears spectacles and has false teeth? **[Ans: $\frac{59}{100}$]**

40. The number of people who play football (F) or basket ball (B) is twice the number of people who play F and B. If n(F)= 9and n(B) = 6, how many play both games?

THE PROBABILITY THEORY

RECALL

◊ The term probability arose from games chances. For example, tossing a coin, rolling a die etc.

◊ Probability is a mathematical measurement of how something is likely to happen.

◊ Probability ranges from **0(zero)** to **1(one)**. Zero means that the event is impossible to happen but **1** means that the event is absolutely certain.

◊ Sample space (S); is a set of all possible outcomes of an experiment. E.g when a coin is tossed, a head (H) or a tail (T) is expected. Therefore S = **{H,T}**.but each out in the sample space is called a **sample point**

◊ Probability = $\frac{\text{number of events (E)}}{\text{number of sample space (S)}} = \frac{n(E)}{n(S)}$

◊ "**And**'' means intersection

◊ "**Or**'' means Union

◊ Mutually exclusive events; are events which cannot occur at the the same time.eg when a coin is tossed ,events head and tail are mutually exclusive. Meaning that if you get a head, you cannot get a tail. this implies that there is no intersection of two events.

Therefore **P(A or B) = P(A ∪ B) = P(A) + P(B)** **P(A ∩ B) = 0**

◊ Independent events; these are events which occur without being affected by each other. For example, a student offering French and history. French is independent from history.

For two independent events A and B,

P(A ∩ B) = P(A) × P(B)

NOTE: **P(A ∪ B) = P(A) + P(B) – P(A ∩ B)**

TRIAL QUESTIONS

1. A fair die is tossed only once and the number which appears on its top face noted. What is the probability of showing

   (i) a number greater than 3 **[Ans: $\frac{1}{2}$]**

   (ii) an odd number or triangle number. **[Ans: $\frac{2}{3}$]**

2. A bag contains 3 black balls 4 blue balls and 2 yellow balls.

   (a) If two balls are picked at random without replacement find the probability that both are of the same colour. **[Ans: $\frac{5}{18}$]**

   (b) How many black balls must be added to the bag so that probability of drawing a black ball should be $\frac{1}{2}$. **[Ans: 3 black balls]**

   (c) How many yellow balls must be added to the bag so that the probability of drawing a black ball should be $\frac{1}{5}$? **[Ans: 6 yellow balls]**

3. The probability that a pupil passes a physics examination is 0.8 and the probability that she will pass in a chemistry examination is 0.6, Find the probability that the pupil will pass in both examinations. **[Ans: $\frac{12}{25}$]**

4. A bag contains four red balls and six black balls. A ball is picked at random from the bag and not replaced. A second ball is then picked. Find the probability that

(i) both balls are black **[Ans: $\frac{1}{3}$]**

(ii) both balls are red **[Ans: $\frac{2}{15}$]**

(iii) the balls are of different colours **[Ans: $\frac{8}{15}$]**

5. There are 5 red and 4 white balls in a bag. Two balls are picked from the bag at random one after another without replacement. Find the probability that:

(a) both are white **[Ans:$\frac{1}{6}$]**

(b) they are of different colours. **[Ans: $\frac{5}{9}$]**

6. The numerals 1, 2 and 3 are written at random to form two and three digit numbers.

(i) Write down all the possible two and three digit numbers that can be formed from the given numerals if a numeral may be used only once in a number to be formed.

(ii) What is the probability that the number formed is a prime number? **[Ans; $\frac{1}{4}$]**

7. A number is packed at random from the set below; 1, 3, 5, 7, 9, 11, 13, 15. What is the probability that a number packed is;

(i) a prime number. **[Ans: $\frac{5}{8}$]**

(ii) divisible by 3 **[Ans: $\frac{3}{8}$]**

8. A box contains 4 black beads and 3 white beads. A bead was drawn and then replaced. Then a second was drawn and put aside, and then another was drawn. What is the probability that all 3 were white? **[Ans: $\frac{3}{49}$]**

9. A two digit number is formed using the numerals 4, 5 and 6 without repeating any numeral.

(a) Write down the possibility space.

(b) Find the probability that the two digit number is a multiple of three. **[Ans: $\frac{1}{3}$]**

10. The numerals 2.3 and 4 are written at random to form one, two and three digit numbers.

(a) If a numeral is used only once in a number to be formed, write down all the possible one, two and three digit numbers that can be formed from the given numerals.

(b) What is the probability that the number formed is divisible by three. **[Ans: $\frac{3}{5}$]**

11. A bag contains 2 red and 4 blue pens. If a pen is picked at random, find the probability that it is;

(i) Red **[Ans: $\frac{1}{3}$]**

(ii) Red or blue **[Ans: 1]**

12. A coin is tossed twice. Find the probability of getting

(a) a head followed by a tail, **[Ans: $\frac{1}{4}$]**

(b) either two heads or two tails. **[Ans: $\frac{1}{2}$]**

13. A die is thrown. Find the probability that the upper face is showing

(a) a number 4 **[Ans: $\frac{1}{6}$]**

(b) a prime number **[Ans: $\frac{1}{2}$]**

(c) neither a 5 nor **[Ans: $\frac{2}{3}$]**

14. A family of Ibrah and Becky wishes to have three children
(a) Write down the possible outcome for the children

(b) Hence find the probability that a family of three children will consist of 2 boys and one girl

(i) in that order, **[Ans: $\frac{1}{8}$]**

(ii) in any order. **[Ans: $\frac{3}{8}$]**

15. A box contains 3 red balls, 7 yellow balls and 2 green balls. A ball is taken at random. What is the probability of it being;

(a) red **[Ans: $\frac{1}{4}$]** (b) yellow **[Ans: $\frac{7}{12}$]**

(c) not red **[Ans: $\frac{3}{4}$]** d) neither red nor green **[Ans: $\frac{7}{12}$]**

16. (a) Calculate the probability that when a die is thrown, the number obtained on the upper face will be a prime number. **[Ans: $\frac{1}{2}$]**

(b) A coin is tossed and a die is thrown. Calculate the probability of obtaining a head on the coin and a prime number on the die. **[Ans: $\frac{1}{2}$]**

17. A bag contains 14 marbles of which 10 are red and 4 are green. Two marbles are picked at random one at a time without replacement. Find the probability of picking marbles of different colours. **[Ans: $\frac{40}{91}$]**

18. Two dice are thrown once. Find the probability that

(i) both dice show an odd number. **[Ans: $\frac{1}{4}$]**

(ii) the sum of the numbers on the dice is nine. **[Ans: $\frac{1}{9}$]**

19. A box contains n red balls and $(n - 8)$ blue ones. The probability of drawing a red ball is $\frac{3}{4}$. Find the number of balls in the box. **[Ans: 16 balls]**

20. In a game, a player throws two fair dice, each with faces numbered 1 to 6. When the two dice show different numbers, the player scores the sum of the two numbers. When the two dice show the same numbers, the player scores twice the sum of the two numbers. Calculate the probability that a player;

(a) Scores 6 in one throw of the two dice. **[Ans: $\frac{1}{9}$]**

(b) Scores a multiple of 4 in one throw of the two dice. **[Ans: $\frac{1}{3}$]**

(c) Scores 3 in each of two successive throws of the two dice **[Ans: $\frac{1}{324}$]**

21. A number is selected at random from the set A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Find the probability that the number selected is:

(i) A triangular number **[Ans: $\frac{2}{5}$]**

(ii) A factor of 72 **[Ans: $\frac{7}{10}$]**

22. (a) From a group of five children, consisting of three girls and two boys, one child is chosen at random. Write down the probability that the child chosen is a girl. **[Ans: $\frac{3}{5}$]**

(b)Two children are chosen from this group of three girls and two boys on another occasion. Calculate the probability that;

(i) They are both boys **[Ans: $\frac{4}{25}$]**

(ii) They are both girls. **[Ans: $\frac{9}{25}$]**

(iii) They are of different sexes. **[Ans: $\frac{4}{25}$]**

23. The table shows the means of transport 180 students used to report back to school.

	Boys	Girls
Car	15	18
Train	5	8
Walk	50	44
Bus	30	10

If a student is chosen at random in the school, what is the probability that;

(a) The student is a boy? **[Ans: $\frac{5}{9}$]**

(b) The student is a girl who travels to school by car? **[Ans: $\frac{1}{10}$]**

(c) The student is a boy who walks to school? **[Ans: $\frac{5}{18}$]**

24. (a) A bag contains white balls and black balls. The probability of choosing a black ball is $\frac{3}{8}$. If the bag contains 32 balls, find;

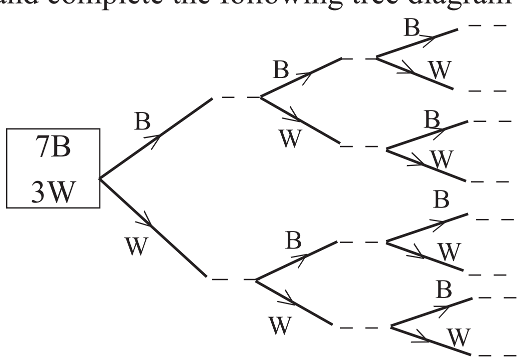
(i) the number of white balls in the bag **[Ans: 20]**

(ii) the probability of picking a white ball **[Ans: $\frac{5}{8}$]**

(b) A basket contains 4 Green and 3 Red apples. Joan picked two apples without replacement, find the probability that they are of the same colour. **[Ans: $\frac{3}{7}$]**

25. A bag contains 8 black marbles and a number of white marbles. The probability of drawing one of the black marbles is $\frac{1}{6}$. How many white marbles are there in the bag and find the probability of picking a white marble. **[Ans: n(W) = 40 mables, P(W) = $\frac{5}{6}$]**

26. A bag contains 7 black and 3 white marbles. Three marbles are chosen at random and in succession, each marble being replaced after it has been taken out of the bag.
Copy and complete the following tree diagram



From your diagram, calculate the probability of choosing;

(a) Three white marbles **[Ans: $\frac{27}{1000}$]**

(b) Atleast one black marble. **[Ans: $\frac{973}{1000}$]**

27. Two dice are thrown and the sum of the two numbers appeared in the upper faces is calculated. Find the probability of this sum being;

(a) 2 **[Ans: $\frac{1}{36}$]**

(b) 7 **[Ans: $\frac{1}{6}$]**

(c) 12 **[Ans: $\frac{1}{36}$]**

(d) a prime number **[Ans: $\frac{4}{9}$]**

28. Twenty cards are numbered from 1 to 20 and then placed in a box. One card is to be chosen at random from the box. Find the probability that the number on the card chosen will be;

(a) greater than 13 **[Ans: $\frac{7}{20}$]**

(b) a multiple of 3 **[Ans: $\frac{3}{10}$]**

29. A coin is tossed 3 times. Find the probability of obtaining;

(i) atmost two Heads **[Ans: $\frac{7}{8}$]**

(ii) two Heads and two Tails **[Ans: $\frac{9}{64}$]**

(iii) atmost one Head or one Tail. **[Ans: $\frac{3}{4}$]**

30. A bag contains 4 red and 3 blue marbles. Three marbles are selected at random one after the other without replacement.

(a) Draw a probability tree to find the possible outcomes.

(b) Find the probability that: (i) all are blue.

(ii) the first two are red and last is blue.

(iii) all are of the same colour.

STATISTICS

RECALL

During the course of study we defined statistics as the branch of mathematics dealing with collection, interpretation, presentation and analysis of data.

Data refers to facts and figures collected for a purpose.

TYPES OF DATA

We categorise data into two i.e,

(i) Un grouped data
(ii) Grouped data

MEASURES OF CENTRAL TENDENCY

These are average values that locate values of a variable in a particular part of the number line.

These include;

(i) **Mean** (means average)
(ii) **Median** (middle number)
(iii) **Mode** (A number / value that appears most)

These are calculated differently but depending on the type of data.

(a) For ungrouped data

(i) Mean = $\frac{\text{sum of all observations}}{\text{number of items}}$

$\bar{x} = \frac{\sum X}{n}$

$\bar{x} \rightarrow$ *mean*

$\sum \rightarrow$ *sum of*

$n \rightarrow$ *number of items*

$x \rightarrow$ *observation*

(ii) Medium;

This refers to the middle number in case data is arranged from the smallest to the biggest. Then, cancel the first number with the last number in that order until you reach the middle number. But if the numbers in the middle are two, get their average.

(iii) Mode; This is the value / number that appears more times than the others in the data.

Rage = Biggest value − smallest value.

Example;

Given that 10, 20, 48, 68, 75, 68, 84, 75, 68, 4, 6 were the marks obtained by S.3 computer students in a mid term test. Use the data to calculate the;

(i) Mean (ii) Medium (iii) Mode (iv) Range

Solution;

(i) Mean = $\frac{\text{sum of all marks}}{\text{number of students}}$

Mean = $\frac{10 + 20 + 48 + 68 + 75 + 68 + 84 + 75 + 68 + 4 + 6}{11}$

Mean = $\frac{526}{11}$ Mean mark = 47.8

(ii) Median arranges in ascending order. (~~4~~, ~~6~~, ~~10~~, ~~20~~, ~~48~~, 68, ~~68~~, ~~68~~, ~~75~~, ~~75~~, ~~84~~)

Median mark = 68

(iii) Mode; The number that appears most is 68 (it appears three times)

(iv) Range = 84 − 4 = 80 units

(b) For grouped data

This will include the following;

Mean	Class interval	Class mark	Quartales
Mode	Class width	Cummulative frequence	Percentiles
Medium	Class boundaries	Assumed mean	

Under this section, data is grouped into **classes** i.e 152 - 154, 155 - 157, 158 - 160 etc and these values are reffered to as **limits**. The limits are classified as <u>lower class limits</u> and <u>upper class limits</u> ie 152 is a lower class limit and 154 is upper class limit for a class 152 − 154.

CLASS MARK

Is the average of the class limits.

Class mark (x) = $\frac{\text{lower class limit + class limit}}{2}$

$= \frac{152 + 154}{2}$

$= 153$

CLASS BOUNDARY

When the data is not given in decimals, we subtruct 0.5 from lower limit and add 0.5 to the upper class limit to form lower class boundaries and upper class boundaries respectively.

e.g

Class	Class boundary
152 - 154	151.5 - 154.5
155 - 157	154.5 - 157.5
158 - 160	157.5 - 160.5

Class interval

= (upper class limit - lower class limit) + 1

= 154 − 152

= 2 + 1 = 3

For data in one decimal place, add 0.1 instead of 1

But, class width = upper class limit − lower class limit

= 154 − 152

class width = 2

OR

Class interval = Upper boundary - lower class boundary ie

154.5 − 151.5 = 3

class interval = 3

Note: When data is given in decimals, we nolonger subtract and add 0.5 to lower and upper class limits respectively to obtain class boundaries in the same order. We subtract and add depending on the number of decimals:

◊ For no decimal place - subtract 0.5 and add 0.5 towards the lower and upper class limits respectively. e.g Given 10 − 14 = 9.5 − 14.5

◊ For one decimal ie 1.0 - 1.4, subtract 0.05 and 0.05 towards the lower and upper class limits respectively.

$(1.0 - 0.05) - (1.4 + 0.05)$

$= 0.95 - 1.45$

◊ For two decimal places eg 2.44 − 2.48, we use 0.005.

◊ For three decimal places, we use 0.0005.

This implies that we add on zero; eg for 4 decimal places, we use 0.00005.

MEAN AND ASSUMED MEAN

Mean of grouped data $= \frac{\sum fx}{\sum f}$	**Assumed mean (A)**	**Mode for grouped data.**
$\sum \rightarrow$ sumation $f \rightarrow$ frequency $x \rightarrow$ class mark	Mean $= A + \frac{\sum fd}{\sum f}$ $d \rightarrow$ deviation $d \rightarrow X - A$ $x \rightarrow$ class mark $f \rightarrow$ frequency	Mode $=$ Lcb $+ \frac{D_1}{D_1 + D_2} x i$ $D_1 \rightarrow$ Highest frequency -frequence before it $D_2 \rightarrow$ Highest frequence - frequency after it. Lcb $\rightarrow$ Lower class boundary of the modal class. $L \rightarrow$ Class interval
Frequency is the number of times a number or a class appears.		

CUMMULATIVE FREQUENCY

Obtained by a continous adding of frequencies to next frequency basing on the first frequency ie;

Class	Frequency	Cummulative frequency	Class boundary
10 - 14	2	2	9.5 -14.5
15 - 19	4	6 (2 + 4)	14.5 - 19.5
20 - 24	8	14 (6 + 8)	19.5 - 24.5
25 - 29	1	15 (14 + 1)	24.5 - 29.5
	$\sum f = 15$		

MEDIAN

Median $=$ Lcb $+ \left(\frac{\frac{N}{2} - cfb}{fm}\right) i$

$N \rightarrow \sum f$

cfb $\rightarrow$ cummulative frequency before the median class

fm $\rightarrow$ frequency of the median class

$i \rightarrow$ class interval

Lcb $\rightarrow$ lower class boundary of the median class

Forexample

Find the median using the above data.

Solution

To find the medium, first obtain $\frac{N}{2}$

$\frac{N}{2} = \frac{15}{2} = 7.5$

Look for where 7.5 would fall in the cummulative frequency column. Therefore 7.5 can not fall in 6 position, but in the next position which is 14.

Then the mediun class is $20 - 24$

L.c.b $= 19.5$

$\frac{N}{2} = \frac{15}{2} = 7.5$

$i = 24.5 - 19.5 = 5$ or width + 1 = class interval

(24 - 20) + 1

4 + 1 = 5

Cummulative frequency before the median class (cfb)

cfb $= 6$

fm $= 8$

From the stated formula;

Median $=$ Lcb $+ \left(\frac{\frac{N}{2} - cfb}{fm}\right) \times i$

$= 19.5 + \left(\frac{7.5 - 6}{8}\right) \times 5$

$= 1.5 + \left(\frac{1.5}{8}\right)$

$= 19.5 + \frac{1.5}{8}$

$= 19.5 + 0.938$

Median = 20.44

ESTIMATING THE QUARTILES FROM GROUPED DATA

We learnt how to estimate the median (the middle quartile) using the formula;

Median $= \text{L.c.b} + \left(\frac{\frac{N}{2} - \text{cfb}}{\text{fm}}\right) \times i$

For lowerquartile (Q1) and the upper quartile (Q3), the formulae are as below.

Lower quartile (Q1) $= \text{L.c.b} + \left(\frac{\frac{N}{4} - \text{cfb}}{\text{fm}}\right) \times i$

Upper quartile (Q3) $= \text{L.c.b} + \left(\frac{\frac{3N}{4} - \text{cfb}}{\text{fm}}\right) \times i$

Interquartile range = Q3 − Q1

GRAPHICAL REPRESENTATION

To find;

(a) Lower quartile, graphically we only use $\left(\frac{N}{4}\right)^{th}$

(b) Upper quartile graphically we only use $\left(\frac{3N}{4}\right)^{th}$

N = total frequency

Graphically, we use a cummulative frequency curve to determine the quartiles.

Example

Class	Frequency	c.f	Class boundary
0 - 9	2	2	-0.5 - 9.5
10 - 19	14	16	9.5 - 19.5
20 - 29	24	50	19.5 - 29.5
30 - 39	12	62	29.5 - 39.5
40 - 49	8	70	39.5 - 49.5
	$\sum f = 60$		

For lower quartile, we use $\left(\frac{N}{4}\right)^{th} = \left(\frac{60}{4}\right)^{th} = 15^{th}$ measure

For upper quartile, we use $\left(\frac{3N}{4}\right)^{th} = \left(\frac{60 \times 3}{4}\right)^{th} = 45^{th}$ measure.

From the graph,

Lower quartile (Q1) = 19

Upper quartile (Q3) = 33

Interquartile ray = 33 − 19

= 14

N.B: Draw a cummulative frequency curve (ogive) obtain a lower quartile and upper quartile bsing on the above measures i.e 15^{th} and 45^{th} respectively.

PERCENTILES

The counts from percentage meaning out of 100.

This is measured basing on the following;

eg 85% means $\frac{85}{100} \times 80 = 68^{th}$

Thus check on this value using a cummulative frequency.

TRIAL QUESTIONS

(1) The following table shows the marks scored by 36 students in a mathematics test.

Marks	Frequency
30 - 39	4
0 - 49	6
50 - 59	3
60 - 69	12
70 - 79	2
80 - 89	5
90 - 99	4

(a) Calculate to 2 decimal places;

(i) Mean mark

(ii) Median mark

(b) Find the probability that a student picked at random scored below 50.

(2) The table below shows the marks obtained by 40 studentd in a mathematics test.

Marks	Number of students
10 - 14	1
15 - 19	4
20 - 24	5
25 - 29	8
30 - 34	12
35 - 39	7
40 - 44	2
45 - 49	1

(a) (i) State the modal class

(ii) Calculate the mode

(b) Draw a histogram to show the marks obtained.

(3) The following are heights of seedling in a nursery bed rounded to the decimal point.

4.7	2.7	2.3	4.6	3.7	2.8	2.9	3.6
4.9	3.9	4.5	3.4	4.2	3.5	1.7	1.1
2.0	3.7	3.3	3.8	3.8	1.8	3.1	3.6
4.1	1.4	1.6	2.1	2.8	2.6	3.3	4.0
3.2	4.3	3.5	2.4	4.4	4.1	2.9	3.2

(a) Draw a frequency distribution table starting with 1.0 - 1.4

(b) (i) State the class interval

(ii) State the modal class.

(c) Calculate:

(i) mean height.

(ii) the median height.

(4) The table below shows marks scored by S.3 students in a Chemistry test in a certain school.

Score	fx
41 - 48	222.5
49 - 56	787.5
57 - 64	1452
65 - 72	753.5
73 - 80	918
81 - 88	253.5

(a) State the modal class.

(b) Calculate;

(i) the mean score

(ii) the median score of the information.

(5) The table below show the marks scored by 75 students in a test.

Mark	10 - 14	15 - 19	20 - 24	25 - 29	30 - 34	35 - 39	40 - 44
Number of students	3	9	14	23	16	8	2

(a) Calculate the; (i) Mean

(ii) Mode

(b) Represent the given data on an Ogive and use it to estimate the median.

(6) The table below shows the mark scored by 50 students in a Biology test.

Mark	Number of students
20 - 29	3
30 - 39	7
40 - 49	16
50 - 59	14
60 - 69	6
70 - 79	3
80 - 89	1

(a) Represent the information on a histogram.

(b) Find the;

(i) Median

(ii) Modal mark

(iii) Mean

(7) Below are marks scored by S.4 students of Luwero High School in the Chemistry mock examinations 2013.

19	20	31	72	40	39	35	32
45	21	13	51	77	15	19	64
10	50	62	40	19	20	27	37
20	53	39	15	42	56	31	12
12	38	31	26	39	63	44	22
58	78	64	18	36	49	54	35
40	38	70	16	29	39	20	21

(a) (i) Form a frequency distribution table consisting of classes of equal width starting with $10 - 19$ as the first class.

(ii) Using 44.5 as your assumed mean, calculate the mean mark of students.

(b) Draw the cummulative frequency curve and use it to estimate the median mark.

(8) Draw a cummulative frequency curve for the following data which shows the marks obtained by 80 form 2 pupils in a school.

Marks	1 - 10	11 - 20	21 - 30	31 - 40	41 - 50	51 - 60	61 - 70	71 - 80	81 - 90	91 - 100
Frequence	3	5	5	9	11	15	14	8	6	4

(a) Draw a cummulative frequency curve and use it to find the following.

(i) the median

(ii) the interquartile range

(iii) the mean.

(9) The table below shows the marks scored by 80 students in a mathematics test.

Marks	Number of students
10 - 19	2
20 - 29	5
30 - 39	8
40 - 49	12
50 - 59	20
60 - 69	12
70 - 79	10
80 - 89	7
90 - 99	4

(a) Using an assumed mean of 54.5, calculate the mean mark.

(b)(i) Draw a histogram for the data.

(ii) Use the histogram to estimate the modal mark.

(10) Below are the heights to the nearest cm of 40 students.

150	170	152	155	169	167	157	158	157
167	164	165	164	163	162	163	158	158
160	160	159	161	161	161	160	160	160
159	162	160	159	160	161	161	156	150

(a) Make a frequency distribution table starting with a class interval 150 - 152

(b) Draw an ogive and use it to estimate the median, interquartile range and 20th percentile height.

(11) The numbers of the eggs collected from a poultry farm for 40 consecutive days were as follows.

138	145	145	157	150	142	154	140
146	135	128	149	164	147	152	138
168	142	135	125	158	135	148	176
146	150	165	144	126	153	136	163
161	156	144	132	176	140	147	130

(a) Construct a frequency distribution table with classes of equal interval width 5, starting from 125 - 129.

(b) Draw a cumulative frequency curve (ogive) and use it to estimate the;

(i) interquartile range

(ii) Median number of eggs.

(12) The table below shows the masses of 40 students to the nearest kg.

Mass (kg)	20 - 29	30 - 39	40 - 49	50 - 59	60 - 69	70 - 79
Number of students	2	m	9	$4m$	8	1

(a) Find the value of m.

(b) Using 50 as the working mean, calculate the mean mass.

(c) Calculate the standard deviation.

(13) The table below shows the marks of 36 candidates in oral examination.

30	31	55	49	56	47
36	41	39	45	39	50
42	43	44	39	46	56
30	48	53	38	50	63
40	54	61	46	56	44
53	60	56	50	62	52

(i) Construct a frequency distribution table having an interval of 6 marks starting with the 30 - 35 class group.

(ii) Draw a cummulative frequency curve and use it to estimate the median mark.

(iii) Calculate the mean.

(14) The table below shows the frequency distribution of marks of 800 candidates who sat a national examination.

Marks (%)	1 - 10	11 - 20	21 - 30	31 - 40	41 - 50	51 - 60	61 - 70	71 - 80	81 - 90	91 - 100
Frequency	30	50	100	150	150	130	90	60	30	10

(a) (i) Construct a cummulative frequency distribution for the data.

(ii) Draw a cumulative frequency curve for the distribution.

(b) Use your graph to estimate;

(i) Median mark

(ii) Percentage number of candidates that failed if the pass mark was 50%.

(iii) Inter quartile range

(c) Calculate the mean mark.

(15)

Marks (%)	30 - 34	35 - 39	40 - 44	45 - 49	50 - 54	55 - 59	60 - 64	65 - 69	70 - 74	75 - 79
Number of students	1	2	4	5	6	9	5	1	5	2

The table above shows the marks obtained in a mathematics examination.

(a) (i) Draw a histogram to represent this data.

(ii) Estimate the mode from the histogram.

(16) Study the table below,

Marks	40	42	44	46	48	50
Frequency	1	3	x	7	3	1

(a) Find the value of x if the mean mark is 45.1.

(b) State the modal Frequency.

(17)

65	48	53	53	60	54	60	59	52
55	54	57	57	56	57	62	56	49
55	56	52	54	54	52	58	61	54

(a) Starting with the class of 48 - 50 and using equal class sizes, draw a frequency distribution table.

(b) State the modal class.

(c) Estimate the mean mark using your table in (a) above.

(d) What percentage of the class scored above the mean mark?

(18) The table below shows the ages in years of 80 people who were allowed to enter a cinema hall.

Ages (years)	15 - 19	20 - 24	25 - 29	30 - 34	35 - 39	40 - 44	45 - 49
Number of people	6	5	8	12	21	18	10

(a) Calculate the mean age.

(b) (i) Draw a cumulative frequency curve (ogive) for the data.

(ii) Use the ogive to estimate the median.

(19) The weight of pupils bags at happy hours primary school in kilograms are given below.

3.3	6.0	4.0	2.5	5.4	5.2	4.0	4.3
3.6	5.2	5.5	7.3	4.9	4.4	4.8	5.4
4.5	4.4	2.2	4.9	3.8	4.4	2.5	3.8
3.0	4.8	5.4	3.0	6.1	6.4	4.5	6.0
6.2	3.8	6.5	3.5	4.5	3.0	4.7	4.5

(i) Starting with a class of 2.0 - 2.4, 2.5 - 2.9 and using equal class intervals, form a frequency distribution table for this data.

(ii) Plot a cumulative frequency curve (ogive) for the data. Use your data to estimate the median weight.

(iii) Calculate the mean weight using an assumed mean of 4.7 correct your answer to one decimal place.

(20) The following table shows the distribution of marks obtained by 50 students.

Marks	45 - 49	50 - 54	55 - 59	60 - 64	65 - 69	70 - 74	75 - 79
Students	3	9	13	15	5	4	1

Calculate; (i) The mean (ii) The mode (iii) The median

(21) The cummulative frequency table below shows the marks obtained by 70 candidates in a mathematics mock exam.

Marks	30 - 39	40 - 49	50 - 59	60 - 69	70 - 79	80 - 89
Cumulative frequency	8	18	38	52	64	70

Use the information in the table above to;

(a) Draw an ogive and use it to estimate

(i) The median.

(ii) Number of students who scored below 50%.

(b) Make a frequency table and find the mean mark using an assumed mean of 54.

(22) In a certain tertiary institution, the marks obtained in mathematics test were as follows.

77	54	32	66	88	66	45	84	64
64	25	76	87	67	49	31	58	41
52	69	54	91	75	71	42	74	70
63	50	65	46	56	58	47	54	68
22	64	5	52	57	77	45	49	51

(a) (i) Construct a grouped frequency distribution table starting with 1 - 10, 11 - 20 etc.

(ii) Calculate the mean mark using 50.5 as assumed mean

(b) Construct a cumulative frequency curve and use the C.F curve to estimate the number of students who scored between 40 and 75 marks.

(23) Copy and complete the table below

Marks	Frequency	Mid-mark(x)	fx
20 - 29	.....	24.5	24.5
30 - 39	2	34.5	......
40 - 49	8	......	......
50 - 59	14	54.5	.....
60 - 69	....	.....	.....
70 - 79	6	74.5	......
80 - 89	2	.....	169.0
	$\sum f = 40$		$\sum fx =$

(b) Use the table to determine the

(i) mean mark

(ii) the median mark.

(24) The mean of the numbers 13, 8, 6, 0, 3, 12, x, 11 and 5 is 7.

Determine;

(i) the value of x

(ii) the median

BEARING

RECALL

Remember the following key issues before attempting any question

◊ All drawings in bearing must be drawn on a graph paper.

◊ All angles in bearing are measured starting from the North in a clockwise direction.

◊ You must know the compass and all the different directions described on it.

◊ A sketch diagram is a must as any answer without a sketch diagram is invalid. It should be drawn using a free hand(you don't need to measure).

◊ Be in position to convert from one metric unit to another. Eg *km* to m or *cm* etc.

◊ Never attempt any question on bearing without a protractor, a ruler, squared paper and a pencil.

◊ Neatness is a must requirement that earns you a bonus mark.

◊ You must know how to calculate speed, distance and time.

◊ The formula for distance = speed × time = S × T. This formula has to be mastered.

◊ Read the question at least two times before starting to attempt any bearing numbers.

TRIAL QUESTIONS

1. A plane flew due west from airstrip P at a speed of 280 $kmhr^{-1}$ for $\frac{3}{4}$ hours before reaching airstrip Q. It then altered its course and flew North-West to airstrip R at 220 *km/hr*. From there it flew on a bearing of 060^0 to airstrip S at 240 *km/hr* for $1\frac{1}{2}$ hours. The total time of flight between the four airstrips was $4\frac{1}{2}$ hours.
   (i) By scale drawing, determine the distance and bearing of P from S. Use a scale of 1 *cm* to represent 50 *km*.
   (ii) Determine the total distance of flight from P to S and hence the average speed for the journey.
   (iii) If the plane flew directly back to P at a speed of 200 *km/h*, determine how long it took to fly back to P.
2. A plane moves from airport A to B, 610 *km* away on a bearing of 010^0, there after it changes its course on a bearing of 217^0 and covers a distance of 500 *km* to C. It then moves 500 *km* eastwards to airport D.
   (a) Using a scale of 1cm to represent 50km, draw an accurate diagram for the whole journey
   (b) Find the bearing and distance of A from D.
   (c) If the plane was using a speed of 400 *km/h*, calculate the time taken for the plane to move from A to D.
3. A plane flew due west from airstrip P at a speed of 280 $kmhr^{-1}$ for $\frac{3}{4}$ hours before reaching airstrip Q. It then altered its course and flew North-West to airstrip R at 220 *km/hr*. From there it flew on a bearing of 060^0 to airstrip S at 240 *km*/hr for $1\frac{1}{2}$ hours. The total time of flight between the four airstrips was $4\frac{1}{2}$ hours.
   (i) By scale drawing, determine the distance and bearing of P from S. Use a scale of 1 *cm* to represent 50 *km*.
   (ii) Determine the total distance of flight from P to S and hence the average speed for the journey.
   (iii) If the plane flew directly back to P at a speed of 200 *km/h*, determine how long it took to fly back to P.
4. A place flies from airport A to airport B, 610 *km* away on a bearing of 010^0. There after, it changes its course and flies on a bearing of 217^0 for a distance of 500 *km* to airport C. It then flies eastwards to airport D, also 500 *km* away from C.
   (a) Using a scale of 1cm represent 50 *km*, draw an accurate diagram for the whole journey.
   (b) Find the bearing and distance of A from D.
   (c) If the plane flies at a speed of 400 *km/hr*, calculate the time it will take to fly directly from d back to A.
5. The bearings of towns A and B from town P are 240^0 and 150^0 respectively. If B is 720 *km* from A on the bearing of 120^0,

(a) Draw a sketch to show the relative positions of the three towns.

(b) Using your sketch in (a) above calculate distance; i) AP (ii) BP

(c) Find the bearing of P from B.

(d) If a plane flies from P to B via A at a speed of 450 *km/h*, find how long it takes.

6. (a) A, B and C are three points on the same level ground. The bearing of C from A is 150^0 and B from A is 060^0. From A, the angle of elevation of the top of a tower at C is 30^0, and from B the angle of elevation of the top of the same tower is 17^0. If the distance BC is 2 *km*. Calculate the;

   (i) the height of the tower

   (ii) distance AC

   (iii) the bearing of B from C.

   (b) Two ships Q and R are sailing towards P. At 1144 hours, ship Q is exactly 120 *km* on bearing of 030^0 from P and ship R is 50km on a bearing of 300^0 from P. At this instant, ship R develops engine troubles and cannot continue with the journey. Ship Q receives a signal from ship R and has to change course and steam straight towards ship R at 50 *km/h*. With out using scale drawing, calculate the time of the day ship Q reaches ship

7. Two aeroplanes P and Q leave an airport at the same time. P flies on a bearing of at 900 *km/h* while Q flies due East at 750 *km/h*.

   (a) Using a scale of 1 *cm* to represent 100 *km*, draw an accurate diagram showing the positions of the two aeroplanes after 1hour 20 minutes.

   (b) From the diagram in (a) above determine:

   (i) the distance between the two aeroplanes

   (ii) bearing of P from Q

   (iii) the bearing of Q from P.

8. Town Q is 400 *km* on a bearing of 080° from town P. Town R is 350 *km* on a bearing of 200^0 from town Q and town S is 250 *km* on a bearing of 210^0 from P.

   (a) Using a scale of 1 *cm* to 50 *km*, draw an accurate diagram to show the relative position of the four towns.

   (b) Find the distance and bearing of S from R.

   (c) A helicopter flies from one round the four towns. Find it's average speed if it takes 4 hours 30 minutes all together.

9. A helicopter flies from Moroto due south for 300 *km*. it then flies on a bearing of 255^0 for 350 *km*. from there it flies on a bearing of 02^0 for 400 *km*.

   (i) Draw an accurate diagram showing the journey of the helicopter using a scale of 1 *cm*: 50 *km*.

   (ii) Use your diagram to find the distance and the bearing of Moroto from the final position of the helicopter.

   (iii) Given that the helicopter flies at a steady speed of 200 *km/hr*, find how long the whole journey took

10. A plane leaves an airport A for B which is 300 *km* away on a bearing of 315^0. At B, the plane changes it's course and flies due south to air port C which is 400 *km* from B. At C, the plane flies on a bearing of 180^0 to airport D 500 *km* from C.

    (a) Using a scale of 1cm to represent 50km, draw an accurate diagram to represent the helicopter's journey.

    (b) Use the diagram to determine;

    (i) The bearing of airport A from airport D.

    (ii) The distance of airport A from airport D in km.

    (c) If the plane was flying at an average speed of 1500 *km/hr*. Find the time it took to reach D given no stop over on the way.

11. A ship leaves a port and sails for 120 *km* on the bearing of 062^0. It then changes direction to a bearing of 160^0 and sails for 200 *km* to an island. Using a scale drawing with 1cm representing 20 *km*, find;

    (a) the distance of the island from the port. (b) the bearing of the port from the island.

    (c) How long would it take a ship to sail directly back to the port at a speed f 20 *km/hr*.

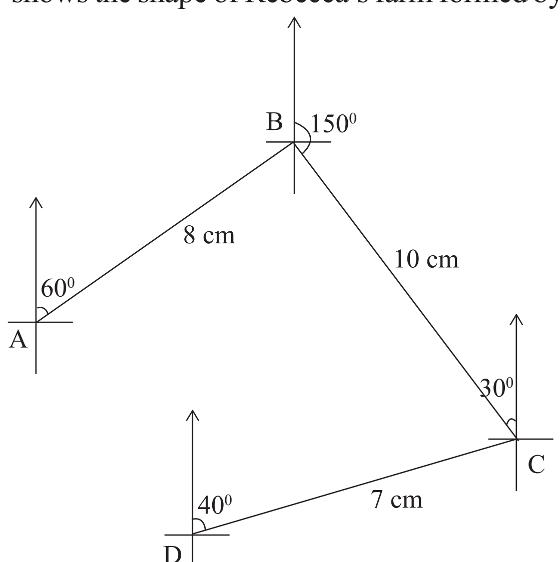
12. A ship leaves a port and sails for 130 *km* on a bearing of 65^0 . It then changes direction on a bearing of 170^0 and sails for 180 *km* to an island. Use a scale drawing with 1 *cm* representing 20 *km* find;
    (a). the distance of the island from the port
    (b) the bearing of the port from the island
    (c) how long would it take the ship to sail directly back to the port at speed of 20 *km*/*hr*.
13. Town B is 300 *km* due west of town **A**. Town C is 450 *km* from town B in the direction N60^0 W and **D** is 600 *km* from C in the direction S30^0W. plane k startsfrom town A and then flies to town B, C and then D at an average speed of 300 *km*/*hr*. plane M flies directly from A to D at an average speed of 200 *km*/*hr*.
    (i) find the time taken by each plane.
    (ii) what is the bearing of town A from town D?.
14. A boat sailed from **P** on a bearing of 060^0 for 40 *km* to **Q**, then sailed on a bearing of 110^0 to point **R** then finally on a bearing of 210^0. If the boat sailed at a speed of 20 kmh^{-1} and from **Q** to **R** it took $2\frac{1}{2}$ hours and then **R** to **S** it was $1\frac{1}{2}$ hours.
    (a) Draw a sketch showing the points **P,Q,R**, and **S**
    (b) Using a scale of 1 *cm* to 10 *km*, draw a scale drawing showing the route of the boat.
    (c) what is the; (i) distance (ii) bearing of S from P
    (d) Find the total time taken for the boat to sail from P to S
15. Leocardia travels from home to a trading centre at a bearing of 030^0, a distance of 300 *km*. from the trading centre, she then travels 600 *km* to the school which is at a bearing of 130° from the trading centre. Assuming that she travels at a constant speed of 20 *km*/*hr* and that 1 *cm* represents 50 *km*.
    (a) draw a sketch to show her journey to school.
    (b) Draw an accurate diagram for the journey
    (c) what is the bearing of; (i) Home from school (ii) school from home
    (d) measure the distance between home and school
    (e) calculate the total time spent on her journey
    (f) if she uses the shortest route to school, calculate the time she takes if she moves at the same speed.
16. Jesca has to travel from town A to town D but because of a very big island between towns A and D, he travels from town A at a bearing of 150^0 a distance of 200 *km* to town B. from town B, he moves at a bearing of 090^0 a distance of 300 *km* to town C. from C he moves to town D which is 400km from C at a bearing of 020^0. Using a scale of 1 *cm* to represent 45 *km*.
    (a). Draw a diagram to show her journey
    (b)Measure, (i) Distance of A from D (ii) Bearing of D from A
    (c) If he took a total time of five hours to travel and assuming that he travelled at a constant speed. Calculate his speed.
17. From the airstrip at Johannesburg, an aeroplane moves at a bearing of 090^0 to Botswana a distance of 200 *km*. it then moves at a bearing of 020^0 a distance of 300 *km* to town A. at town A, it rests for two hours and then moves at a distance of 170^0 to Kigali which is 500 *km* from town A. (use scale 1cm: 50 *km*)
    (a) draw a diagram to show the journey of the plane
    (b) distance between Johannesburg and Kigali
    (c)Bearing of Johannesburg from Kigali.
    (d)Time spent, if the plane travelled at a constant speed of 80 *km*/*hr*.
18. A cargo ship sails from island A to island B at a bearing of 250^0 and a distance of 250 *km* from island B. It moves N30^0W a distance of 550 *km* to island C. If 1cm represents 60 *km*.
    (a) Draw a diagram to illustrate the ship's journey. (b) Distance between island C and A
    (c) If the shop took 2 hours to travel from island A to B and $3\frac{1}{2}$ hours to travel from B to C. calculate the respective speeds.
    (d) Bearing of (i) Island C from A (ii) Island A from C

19. In the last year's motorcar sport competitions, motorcars started from town P and moved at a bearing of 060^0 for a distance of 120 *km* to town Q. They then moved in a direction described as $N80^0E$ a distance of 150 *km* to town R. Then they moved at a bearing of 230^0 for 300 *km* to town S. If 1 *cm* represents 20 *km*;
    (a) Draw a diagram to illustrate their journey
    (b) Bearing of P from S
    (c) If a car was to take a short cut from Q to S, what distance did it go through.
    (d) What is the bearing of Q from S.

20. Dorcus started running from station L at a bearing of 330^0 for a distance of 10 *km* to station M. she then moved in the Easter direction at a speed of 5 *km*/*hr* for 2 hours to junction K. she then turned through a bearing of 072^0 and moved for 15 *km* to station N. she then moved to station P which is 18km south of station L. If 1 *cm* represents 2 *km*,
    (a) Draw a diagram to show her journey
    (b) distance and bearing of L from P
    (c) distance and bearing of L from N
    (d) Total time spent if she was running at a speed of 2 *km*/*hr*.

21. Points P, Q, R and S are oriented on the play ground such that point B is at a bearing of 000^0 from point P, point S is at a bearing of 340^0 from point P, point R is at a bearing of 070^0 from point Q. if distance PS is 100 *m*, PQ is 40 *m* and QR is 30 *m* and that 1cm represents 7 *m*.
    (a) Draw an accurate digram to show the relationship between the points.
    (b) bearing of S from Q and R.

22. The figure below shows the shape of Rebecca's farm formed by poles A, B, C and D and 1 *cm* represents 100 *m*



If the owner wants to add a pole between B and C such that BK : KC = 1 : 3 where K is the pole point.
    (a) Draw an accurate diagram including point K to represent the farm.
    (b) Measure distance AK and KD in metres
    (c) Determine the bearing of; (i) K from A (ii) A from K (iii). K from D.
    (d) Determine distance KD in metres and bearing of A from D

FACTORIZATION

RECALL

◊ This is the process of creating a list of factors. When these are multiplied together, will produce a desired quantity.

◊ In general, it is a process of breaking up an expression into its separate factors.

◊ The easiest type of factor to find is a single term factor common to all terms of the expression and this is usually obvious.

For example;

Factorise $2p - 2q$.

Solution

◊ Realise that 2 is common through out, then factorise it out and introduce brackets, i.e; $2(p - q)$
Note: Factors of $2p - 2q$ are 2 and $(p - q)$.

◊ Therefore when factorizing an expression, first see if there is a common factor of each term.

◊ Group together all terms which have a common factor and then factorize each term before finding a bracket which is one of the factors of the original expression.

Example 1

Factorize $4x - 4y + 7x - 7y$

Solution

Collect together the terms which have a common factor.

$4x - 4y + 7x - 7y$

$4(x - y) + 7(x - y)$

$(x - y)(4 + 7)$

$= 11(x - y)$

OR

Collect like terms

$4x + 7x - 4y - 7y$

$11x - 11y$

$11(x - y)$

Example 2

Factorize the following expressions

(a) $33\,km - 44\,kr$

Find the H.C.F

HCF = $11k$

$11k \times 3m - 11k \times 4r$

factor out $11k$ because its common

$\underline{11k(3m - 4r)}$

(b) $7k^2 - k$

HCF is k

$k \times 7k - k \times 1$

k is common

$\underline{k(7k - 1)}$

(c) $au + av - u - v$

solution

collect like terms

$au - u + av - v$

$u(a - 1) + v(a - 1)$

but $(a - 1)$ is common, so factor it out

$(a - 1)(u + v)$

(d) $fm + fn + gm + gn$

Solution

collect like terms

$fm + gm + fn + gn$

$m(f + g) + n(f + g)$

$(f + g)$ is common, so

$(f + g)(m + n)$

Example 3

Factorize the following

(a) $a^2p + a^2p + a^2q$, a^2 is common

pq^2 and p^2q, pq is common

$a^2p + a^2q - pq^2 - p^2q$

$a^2(p + q) - pq(q + p)$

but $(p + q)$ is common

$= (p + q)(a^2 - pq)$

(b) $1 + y^2 + xy(1 + y^2)$

Realise that $(1 + y^2)$ is common then factor out

$= (1 + y^2)(1 + xy)$

(c) $xp - 4xp^2 - z + 4zp$

$xp(1 - 4p) - z(1 - 4p)$

$= (1 - 4p)(xp - z)$

DIFFERENCE BETWEEN TWO SQUARES

◊ The product of the sum and the difference of two numbers is equal to the difference of their squares.

◊ $a^2 - b^2 = (a + b)(a - b)$. This is a fundamental identity at o'level.

Example 1

Factorize the following

(i) $x^2 - 9$

solution

$(x)^2 - (3)^2$

But $(a^2 - b^2) = (a + b)(a - b)$

$(x)^2 - (3)^2 = (x + 3)(x - 3)$

(ii) $p^2 - 36$

solution

$(p)^2 - (6)^2$

But $a^2 - b^2 = (a + b)(a - b)$

$p^2 - 6^2 = (p + 6)(p - 6)$

(iii) $p^2 - 4q^2$

solution

$p^2 - 2^2q$

$p^2 - (2q)^2$

But $a^2 - b^2 = (a + b)(a - b)$

$p^2 - (2q)^2 = (p + 2q)(p - 2q)$

(iv) $4y^2 - 9a^2b^2$

Solution

$4y^2 - 9a^2b^2$

$2^2y^2 - 3^2a^2b^2$

$(2y)^2 - (3ab)^2$

but $a^2 - b^2 = (a + b)(a - b)$

$(2y)^2 - (3ab)^2 = (2y + 3ab)(2y - 3ab)$

(v) $(a + b)^2 - c^2$

Solution

But $a^2 - b^2 = (a + b)(a - b)$

$(a + b)^2 - c^2 = (a + b + c)(a + b - c)$

(vi) $8x^2 - 18$

Solution

$2(4x^2 - 9)$

$2(2^2x^2 - 3^2)$

$2((2x)^2 - 3^2)$

But $(2x)^2 - 3^2 = (2x + 3)(2x - 3)$

$2[(2x + 3)(2x - 3)]$

$2(2x + 3)(2x - 3)$

Example 2

Factorise the following

(a) $27a^2 - 12$

Solution

$3(9a^2 - 4)$

$3(3^2a^2 - 2^2)$

$3(3a)^2 - 2^2)$

$3[(3a + 2)(3a - 2)]$

$3(3a + 2)(3a - 2)$

(b) $a^2b^2c^2 = 1$

solution

$(abc)^2 - 1^2$

$= (abc + 1)(abc - 1)$

(c) $48a^3b^2 - 27ab^4$

solution

$3ab^2(16a^2 - 9b^2)$

$3ab^2(16a^2 - 9b^2)$

But $3ab^2(4^2a^2 - 3^2b^2)$

$3ab^2[(4a)^2 - (3a)^2]$

$3ab^2[(4a + 3b)(4a - 3b)]$

$3ab^2(4a + 3b)(4a - 3b)$

Example 3

Without using calculators or tables, evaluate;

(i) $7.46^2 - 2.54^2$

$a^2 - b^2 = (a + b)(a - b)$

$7.46^2 - 2.54^2 = (7.46 + 2.54)(7.46 - 2.54)$

$= (10)(4.92)$

$= 49.2$

(ii) $5.2 \times (3.75^2 - 1.25^2)$

solution

$a^2 - b^2 = (a + b)(a - b)$

$3.75^2 - 1.25^2 = (3.75 + 1.25)(3.75 - 1.25)$

$5.2 \times (3.75 + 1.25)(3.75 - 1.25)$

$5.2 \times (5)(2.5)$

$\frac{52}{10} \times \frac{5}{1} \times \frac{25}{10}$

$= 65$

(iii) $\frac{6.51^2 - 2.81^2}{0.932 \times 4}$

solution

$$\frac{(6.51 + 2.81)(6.51 - 2.81)}{0.932 \times 4}$$

$$\frac{(9.32)\ (3.70)}{\frac{932}{1000} \times 4} = \frac{\frac{932}{100}\ \frac{37}{10}}{\frac{932 \times 4}{1000}}$$

$$= \frac{932 \times 37}{1000} \div \frac{932 \times 4}{1000}$$

$$= \frac{932 \times 37}{1000} \times \frac{1000}{932 \times 4}$$

$$= \frac{37}{4}$$

FACTORIZATION OF A QUADRATIC EXPRESSION.

◊ Remember a quadratic expression is of form $ax^2 + bx + c$ but when we equate the expression to zero ie; $ax^2 + bx + c = 0$, it becomes an equation because of the equal sign.

◊ When factorizing the above quadratic expression, we find two numbers such that their sum is b and their product is ac

$$\underbrace{ax^2 + bx + c}_{ac}$$

Example 1

Factorise $x^2 - 4x + 3$

Look for numbers such that their sum is $^-4$and their product is 3.

$$\underbrace{x^2 - 4x + 3}_{ac\, =\, 1 \times 3 = 3}$$

The numbers are $^-1$ and $^-3$

$x^2 - x - 3x + 3$

$x(x - 1) - 3(x - 1)$

$(x - 1)(x - 3)$

$x^2 - 4x + 3 = (x - 1)\ (x - 3)$

Example 2

$2y^2 + y - 6$

solution

$\underbrace{2y^2 + y - 6}$

$2 \times 6 = {}^-12$

look for two numbers that when multiplied give $^-12$ but when added, give 1. the numbers are 4 and $^-3$

$2y^2 + 4y - 3y - 6$

$2y(y + 2) - 3(y + 2)$

$(y + 2)\ (2y - 3)$

$2y^2 + y - 6 = (y + 2)(2y - 3)$

Example 3: Factorize: $(p + 2q)^2 - 4(p + 2q) + 3$

Let $p + 2q$ be y (becomes $p + 2q$ is common)

$y^2 - 4y + 3$. Then,

factorize $\underbrace{y^2 - 4y + 3}_{3}$ factors are $^-1$, $^-3$

$y^2 - y - 3y + 3$

$y(y - 1) - 3(y - 1)$

$(y - 1)(y - 3)$ but $y = p + 2q$

$(p + 2q - 1)(p + 2q - 3)$

Example 4: Factorize $20x^2y^2 + xy - 1$

Solution

$20x^2y^2 + xy - 1$

$20(xy)^2 + xy - 1$

let $xy = n$

$\underbrace{20n^2 + n - 1}_{^-20}$ factors are 5 and $^-4$

$20n^2 + 5n - 4n - 1$

$5n(4n + 1) - (4n + 1)$

$(4n + 1)(5n - 1)$

but $n = xy$

$(4xy + 1)(5xy - 1)$

TRIAL QUESTIONS

Without using tables or calculators (Use factorization)

1. Find the values of;

   (a) $0.1 \times 45 + 0.1 \times 55$ **[Ans: 10]**

   (b) $500 \times 4 + 4 \times 600$ **[Ans: 4400]**

   (c) $(21.73 \times 14.6) + (2173 \times 5.4)$ **[Ans: 434.6]**

   (d) $(617 \times 793) + (786 \times 793) + (597 \times 793)$ **[Ans: 1586000]**

   (e) $(75 \times 130 - 75 \times 30)$ **[Ans: 7500]**

   (d) $\frac{(98 \times 154 - 98 \times 54}{(10 \times 6.3) + 10 \times 3.7}$ **[Ans: 98]**

2. Factorise the following expressions;

   (a) $8xy + 4xy^2$ **[Ans: $4xy(2 + y)$]**

   (b) $8x^2yz + 4xy^2z + 12xyz^2$ **[Ans: $4xyz(2x + y + 3z)$]**

   (c) $P + \frac{PRT}{100}$ **[Ans: $P(I + \frac{RT}{100})$]**

   (d) $\pi r^2h + \frac{2}{3}\pi r^3$ **[Ans: $\pi r^2(h + \frac{2}{3}r)$]**

3. Factorize the following

   (a) $(a - b)^2 - 2(a - b)$ **[Ans: $(a - b)\ (a - b + 2)$]**

   (b) $p(x - 1) + (x - 1)$ **[Ans: $(x - 1)\ (p + 1)$]**

   (c) $2a^2(a - x) + ax(a - x)$ **[Ans: $a(a - x)\ (2a + x)$]**

   (d) $a(a - b)$ - $3(a - b)$ **[Ans: $(a - b)\ (a - 3)$]**

   (e) $x(p^2 + 2pq + q^2) - 3y(p + q)^2$ **[Ans: $(p + q)^2(x - 3y)$]**

   (f) $(x - 1)^2 - yx + y$ **[Ans: $(x - 1)\ (x - y - 1)$]**

   (g) $x(p + w + z) + y(p + w + z)$ **[Ans: $p + w + z)\ (x + y)$]**

4. Factorize the following expressions

   (a) $8y^2 - 32$ **[Ans: $8(y + 2)\ (y - 2)$]**

   (b) $144p^2 - 25$ **[Ans: $(12p + 5)\ (12p - 5)$]**

   (c) $2x^2 - 18$ **[Ans: $2(x + 9)\ (x - 9)$]**

   (d) $5x^2 - 125$ **[Ans: $5(x + 5)\ (x - 5)$]**

   (e) $9q^2 - 9$ **[Ans: $9(x + 1)\ (x - 1)$]**

   (f) $3x^4 - 3$ **[Ans: $2(x + 1)\ (x - 1)\ (x^2 + 1)$]**

   (g) $25p^2 - 36$ **[Ans: $5p + 6)\ (5p - 6)$]**

   (h) $x^2 - 16$ **[Ans: $(x + 4)\ (x - 4)$]**

   (i) $9 - 16p^2q^2$ **[Ans: $(3 + 4pq)\ (3 - 4pq)$]**

   (j) $16x^2 - 64$ **[Ans: $(16(x + 2)\ (x - 2)$]**

   (k) $25p^2 - 49q^2$ **[Ans: $(5p + 7q)\ (5p - 7q)$]**

   (l) $m^4 - n^4$ **[Ans: $(m^2 + n^2)\ (m + n)\ (m - \mathrm{n})$]**

   (m) $45 - 20k^2$ **[Ans: $5(3 + 2k)\ (3 - 2k)$]**

   (n) $8x^2 - 18y^2$ **[Ans: $2(2x + 3y)\ (2x - 3y)$]**

   (o) $q^4 - 625$ **[Ans: $(q^2 + 25)\ (q + 5)\ (q - 5)$]**

   (p) $3(x - 4)^2 - 12$ **[Ans: $(3(x - 2)\ (x - 6)$]**

   (q) $3p^3 - 12pq^2$ **[Ans: $3p(p + 2q)\ (p - 2q)$]**

5. Use factorization to find the values of

   (a) $416^2 - 284^2$ **[Ans: 92400]**

   (b) $90^2 - 10^2$ **[Ans: 8000]**

   (c) $\frac{6.51^2 - 2.81^2}{0.932 \times 74}$ **[Ans: $\frac{1}{2}$]**

6. Factorize the following;

   (a) $x^2 - x - 156$ **[Ans: $(x + 12)(x - 13)$]**

   (b) $x^2 - 3x$ **[Ans: $x(x - 3)$]**

   (c) $x^2 + 9x - 36$ **[Ans: $(x - 3)(x + 12)$]**

   (d) $x^2 - x - 12$ **[Ans: $(x + 3)(x - 4)$]**

   (e) $2x^2 + 9x - 11$ **[Ans: $(x - 1)(2x + 11)$]**

   (f) $^-2t^2 + t + 6$ **[Ans: $(2t + 3)(2 - t)$]**

   (g) $9n^2 + 18n - 7$ **[Ans: $(3n - 1)(3n + 7)$]**

   (h) $6x^2 - x - 12$ **[Ans: $(3x + 4)(2x - 3)$]**

   (i) $36 - (x^2 + 2xy + y^2)$ **[Ans: $(6 + x + y)(6 - x - y)$]**

   (j) $a^2 - 3a(b + 2c) + 2(b + 2c)^2$ **[Ans: $(a - 2b - 4c)(a - b - 2c)$]**

   k) $2 + 2x^2 + xy + x^2y$ **[Ans: $(1 + x^2)(2 + xy)$]**

   (j) $9(x + 2)^2 - 12(x + 2) + 4$ **[Ans: $(3(x + 2) - 2)(3(x + 2) - 2)$]**

ALGEBRA OF FRACTIONS AND EQUATIONS

RECALL

◊ This involves dividing the numerator by the denominator.

◊ This involves creating factors of a denominator and the numerator where need be and then cancel the factors of the same kind.

◊ When operations (adding, subtraction) are involved, then find the LCM of the denominators and proceed normally to simplify. (review operations on fractions).

Example: Simplify the following fractions;

(a) $\frac{x+1}{x^2-1}$

Solution

$\frac{x+1}{x^2-1}$, But $\frac{x^2+1}{x^2-1^2} = (x+1)(x-1)$ (creating factors of the denominator)

$\frac{(x+1)}{(x+1)(x-1)}$ (cancelling the same factors)

$= \frac{1}{x-1}$

(b) $\frac{x}{1-x} - \frac{x}{1+x} + \frac{2x}{1-x^2}$

solution

$1 - x^2 = (1-x)(1+x)$

$\frac{x(1+x) - x(1-x) + 2x}{(1-x)(1+x)}$

$\frac{x + x^2 - x + x^2 + 2x}{(1-x)(1+x)}$

$\frac{2x^2+2x}{(1-x)(1+x)} = \frac{2x(x+1)}{(1-x)(1+x)} = \frac{2x}{1-x}$

◊ But when an expression is equated to zero, then it becomes an equation which requires to be solved.

Equations

◊ An equation is a statement in algebra which states that the two given expressions are the same. In between the two expressions there exists an equal sign.

e.g, $x + 9 = 10$, $\frac{x^2-x+2}{4(x-2)^2} = 0$, But equations with one unknown of first degree are called linear equations e.g $x + 3 = 5$.

Example: Solve for x: $\frac{5}{x} + \frac{1}{3} = \frac{8}{9}$

Solution

LCM of x, 3 and 9 is $9x$, Then multiply $9x$ throughout on both sides.

$\frac{5}{\cancel{x}} \times 9\cancel{x} + \frac{1}{3} \times 9x = \frac{8}{9} \times 9x$

$45 + 3x = 8x$

$45 = 5x$

$\frac{45}{5} = \frac{5x}{5}$

$x = 9$

TRIAL QUESTIONS

1. $\frac{2x+5}{x-1} - \frac{x+4}{x+3} = 1$ **[Ans: $x = \frac{^-11}{3}$]**
2. $\frac{4}{3x} - 4 + \frac{2}{x} = \frac{4}{5}$ **[Ans: $x = \frac{25}{36}$]**
3. $\frac{3}{x+3} - \frac{1}{5x-2} = 0$ **[Ans: $x = \frac{9}{14}$]**

4. $\frac{2m-5}{5} + \frac{m-4}{3} = \frac{m+14}{30}$ **[Ans: $m = 4$]**

5. $\frac{p-5}{2} + \frac{p+2}{3} = \frac{4p-1}{3}$ **[Ans: $p = {}^-3$]**

6. Mr. Zziwa is three times as old as his daughter. 15 years ago, the father was eighteen times as old s his daughter. Find the present age of;
   (i) the daughter **[Ans: 17 years]**
   (ii) Mr. Zziwa **[Ans: 51 years]**

7. SOlve the following equations
   (a) $\frac{4x-1}{2} + \frac{x+3}{3} + \frac{x+7}{6} = 0$ **[Ans: $\frac{{}^-2}{3}$]**
   (b) $\frac{1}{x} + \frac{3}{2x-7} = 0$ **[Ans: $\frac{7}{5}$]**
   (c) $\frac{4}{5}(k+1) - \frac{k}{3} = 2\frac{2}{3}$ **[Ans: $k = 4$]**
   (d) $\frac{y+3}{2\frac{1}{2}} + \frac{y-2}{3\frac{1}{3}} = \frac{3}{5}$ **[Ans: $y = 0$]**
   (e) $\frac{1}{6}(x+2)$ - $\frac{1}{9}(2x-3) = \frac{1}{12}(1-3x)$ **[Ans: $x = {}^-3$]**
   (f) $\frac{3}{4}(2p-5) + \frac{1}{6}(p+7) = \frac{2}{3}(p-2)$ **[Ans: $p = \frac{5}{4}$]**
   (g) $\frac{x}{1-x}$ - $\frac{x}{1+x} + \frac{2x}{1-x^2} = 1$ **[Ans: $x = \frac{1}{3}$]**

8. Muzungu is three times as old as his son. In 12 years time, he will be twice as old as his son.
   (i) How old is his son now? **[Ans: 12 years]**
   (ii) How old is the father now? **[Ans: 36 years]**

9. Rebecca is five years younger then her sister Muwanguzi and Annet is twice as old as Rebecca. The sum of their ages is 49. Find;
   (i) Muwanguzi's age **[Ans: 16 years]**
   (ii) Rebecca's age **[Ans: 11 years]**
   (iii) Annet's age **[Ans: 22 years]**

10. Solve the following equations
    (a) $\frac{3y}{2} - \frac{y}{3} = \frac{5(y-4)}{6}$ **[Ans: $y = {}^-10$]**
    (b) $\frac{1}{3}(t-2) = \frac{1}{4}(t+1)$ **[Ans: $t = 11$]**
    (c) $\frac{q+3}{5} = 8 - \frac{q-1}{4}$ **[Ans: $q = 17$]**
    (d) $\frac{x}{2}$ - $\frac{x+1}{3} = \frac{1}{3}$ **[Ans: $x = 4$]**

11. After marking a test, Recheal got twice as many marks as Nakato and Christine got 8 marks more than Recheal. The total of all their marks was 223. What is;
    (i) Nakato's marks? **[Ans: 43 years]**
    (ii) Recheal's marks? **[Ans: 86 years]**
    (iii) Christine's marks? **[Ans: 94 years]**

12. Nyangoma travels 6 *km* to work. He walks parts of the way at 5 *km/hr* and gets a bus the rest of the way at 40 **km/hr**. The whole journey takes 30 minutes. How far does he walk? **[Ans: 2 *km*]**

13. Winniefred has twice as much money as her sister. Together we have shs 60,000. How much money does Winniefred's sister have? **[Ans: 20,000]**

14. Kaguta is twice as old as his son Kasolo. Ten years ago he was three times as old as his son. How old is the son at present? **[Ans: 20 years]**

15. Angle A of triangle ABC is double the size of angle B which is three times the size of angle C. Find the size of the angles A, B, C. **[Ans: A = 108^0, B = 54^0, C = 18^0]**

SIMULTANEOUS EQUATIONS

RECALL

Methods that are majorly used to solve simultaneous equations.

◊ Elimination method

◊ Substitution method

◊ Inverse method

◊ Matrix method

TRIAL QUESTIONS

1. (a) Solve the simultaneous equations below:

$$5x + 2y = 5$$
$$3x - 0.2y = 10$$

[Ans: $x = \frac{^-9}{5}, y = 7$]

(b) In the market Ndgire buys 4 *kg* of tomatoes and 2 *kg* of potatoes for shs 3,800 = Kiwanuka buys 5 *kg* of tomatoes and 3 *kg* of potatoes for shs 5,000. Find

(i) the cost of 1 *kg* of tomatoes and 1 *kg* of potatoes **[Ans: 700, 500]**

(ii) if Claudia wants to buy 10 *kg* of tomatoes and 15 *kg* of potatoes find how much she will spend. **[Ans: 14,500]**

2. Solve the simultaneous equations

(i) $2a + b = 10$ and $a - 5b = {}^-6$ [**Ans**: $a = 4$, $b = 2$] (ii) $2x + y = {}^-5$ and $y - 4x = 13$ **[Ans: $x = {}^-3, y = 1$]**

(iii) $y + 5x = 7$ and $2x - 4y = 7$ **[Ans: $x = \frac{35}{22}, y = \frac{^-21}{22}$]**

(iv) $2x + 5y = 12$ and ${}^-5 + x = {}^-2y$ **[Ans: $x = \frac{^-1}{9}, y = \frac{22}{9}$]** (v) $2a + 3b = 13$ and $3a - 4b = 11$ **[Ans: $a = 5, b = 1$]**

3. (a) Use matrix method to solve the following pair of simultaneous equations.

$$3a + 4b = 8$$
$$a + 2b - 3 = 0$$

[Ans: $a = 2, b = \frac{1}{2}$]

(b) A grocery sells two kinds of meat products A and B . Okot bought 4 *kg* of A and 6 *kg* of B paying a total of shs. 5280. Namusisi bought 5 *kg* of A and 3 *kg* of B at a total cost of shs. 4440.

(i) Write down two equations to describe okot's and Namusisi's purchases.

(ii) By combining the two equations in matrix form, determine the cost of a *kg* of each meat product.

(iii) How much would Mugisha pay for 6 *kg* of A and 5 *kg* of B? **[Hint: Refer to No.1(b)]**

(c) Use matrix method to solve the equations

$$3a - 5b + 9 = 0$$
$$5a + 2b = 16$$

[Ans: $a = 2, b = 3$]

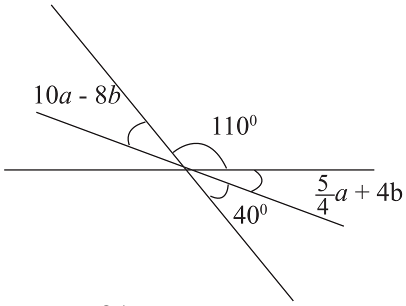
4. Three kilograms of sugar and four kilograms of rice cost shs 22,400 while two kilograms of rice and five kilograms of sugar cost shs 21,000.

(i) Find the cost of a kilogram of sugar and a kilogram of rice.

(ii) How much will Mr. Musoke spend if he buys six kilograms of rice and nine kilograms of sugar?

5. Solve for a and b in the diagram below.

[Ans: $a = 12$, $b = 10$]



6. $p + q = 21$
   $p - q = 3$ **[Ans: $p = 12$, $q = 9$]**

(7) $3x + y = 7$
   $x + y = 3$ **[Ans; $x = 2$, $y = 1$]**

8. $x + y = 10$
   $x - y = {}^{-}4$ **[Ans; $x = 3$, $y = 7$]**

(9) $2p - q = 5$
   $3p + q = 5$ **[Ans: $p = 2$, $q = {}^{-}1$]**

10. $3x + y = 1$
   $4x - y = 6$ **[Ans: $x = 1$, $y = {}^{-}2$]**

(11) $5x - 2y = 7$
   $3x - 2y = 1$ **[Ans: $x = 3$, $y = 4$]**

12. $4x - 3y = 23$
   $4x + 3y = 17$ **[Ans: $x = 5$, $y = {}^{-}1$]**

(13) $s - 3r = 1$
   $s - 5r = {}^{-}1$ **[Ans: $s = 4$, $r = 1$]**

14. $x - 3y = 0$
   $2x - y = 15$ **[Ans: $x = 9$, $y = 3$]**

(15) $3x + 3y = 7$
   $6x - 6y = 7$ **[Ans: $x = \frac{7}{4}$, $y = \frac{7}{12}$]**

16. $5x - 3y = 4$
   $3x + y = 1$ **[Ans: $x = \frac{1}{2}$, $y = \frac{{}^{-}1}{2}$]**

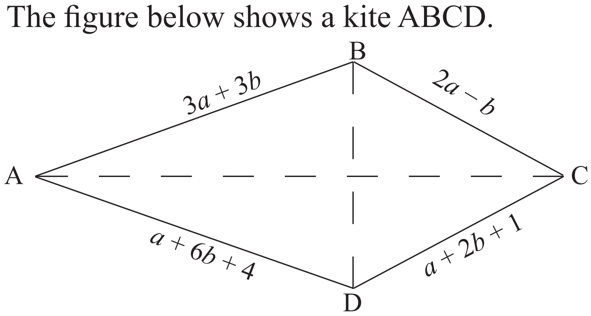
(17) $a - b + 1 = 0$
   $3a = 2b$ **[Ans: $a = 2$, $b = 3$]**

18. $2a + 9b = 3$
   $a + 2b = 5$

(19) $5x + 5y = 3$
   $3y - x = 0.2$ **[Ans: $x = \frac{2}{5}$, $y = \frac{1}{5}$]**

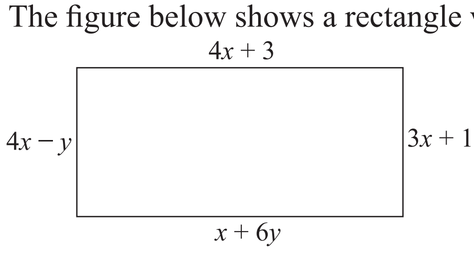
20. $7y - 2x = 0$
   $3y - 2x = 1$ **[Ans: $x = \frac{{}^{-}7}{8}$, $y = \frac{1}{4}$]**

21. The figure below shows a kite ABCD.



Find the values of a and b
[Ans: $a = 3$, $b = \frac{2}{3}$]

22. The figure below shows a rectangle whose dimensions are given in *cm*.



Find the value of x and y and the area of the rectangle.
[Ans: $x = 3$, $y = 2$]

23. Solve the simultaneous equations;
   $y = x + 1$ **[Ans: $x = \frac{{}^{-}8}{5}$, $y = \frac{{}^{-}3}{5}$]**
   $x^2 + 4y^2 = 4$

MATRICES

TRIAL QUESTIONS

1. Given the matrices $A = \begin{pmatrix} 2 & ^-2 \\ 0 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 3 & 2 \\ 4 & ^-1 \end{pmatrix}$ $C = \begin{pmatrix} 5 & ^-4 \\ ^-1 & 2 \end{pmatrix}$. Determine
   (a) AB + C (b) The determinant of (AB + C)
2. Given that $(1 \ \ 3)\begin{pmatrix} 4 & y \\ x & 2 \end{pmatrix} = (7 \ \ 7)$. Find the values of x and y
3. Use matrices to solve the pair of simultaneous equations
   (a) $x - 3y = 9$, $7x + 2y = 17$
   (b) $2y - 4x + 2 = 0$, $3x - 2y = 5$
   (c) $3x - 5y + 9 = 0$, $2y - 16 = -5x$
4. (a) Given that $A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$. Find A^2
   (b) Given that $A = \begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & ^-1 \\ 2 & 3 \end{pmatrix}$ and C = AB, find C^{-1}
5. Given that; $\begin{pmatrix} 3 & a \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 4 & b \\ c & 1 \end{pmatrix} = \begin{pmatrix} d & ^-4 \\ 12 & 2 \end{pmatrix}$ Find the values of a, b, c and d.
6. A news paper vendor supplied copies of "The New Vision", "Red Pepper", "Etop", and "The Monitor" to 4 offices A, B, C , and D once a week as shown in the table below.

	A	B	C	D
Etop	4	2	0	1
New Vision	2	0	1	3
Monitor	3	1	5	1
Red paper	2	1	1	3

   The prices per copy of The New Vision, Red Pepper, Monitor and Etop are sh. 1,000, 1,200 , 1,000 and 800 respectively,
   (a) Write down a 4 × 4 matrix (s) for numbers of copies supplied per week.
   (b) Write down a 1 × 4 cost matrix (C).
   (c) If the vendor supplied the same numbers of copies for 3 weeks , write down a 4 × 4 matrix (T) for the copies supplied per office.
   (d) Find by suitably multiplying any of the two matrices above, how much each office pays the vendor after the 3 weeks.
7. (a). Given that $A = \begin{pmatrix} 6 & ^-3 \\ 5 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} ^-2 & 3 \\ 4 & 8 \end{pmatrix}$.Find $(3A^{-1})B$
8. Four secondary schools football teams of Kibuli S.S, Lubiri S.S Mvara S.S and Nagaalama S.S qualified for a football tournament, which was played in two round with other teams.
   In the first round:
   Kibuli S.S won two matches, drew three and lost one match.
   Lubiri S.S won one match, drew two and lost three matches.
   Mvara S.S won four matches, drew one and lost one match.
   Nagaalama S.S won three matches, drew two and lost one match.
   In the second round:
   Kibuli S.S won three matches and lost three matches
   Lubiri S.S won two matches drew one and lost three matches.
   Nvara S.S won three matches drew two and lost one match.
   Naggaalama S.S won one match, drew one and lost four matches.
   (a) Write down;
   (i) a 4 × 3 matrix to show the performance of the four teams in each of the two rounds

(ii) a 4 × 3 matrix which shows the overall performance of the teams in the two rounds

(b) If three points are awarded for a win, one point for a draw and no point for a loss, use matrix multiplication to determine the winner of the tournament.

(d) Given that MTN donated sh. 2,070,000 to be shared by the four teams according to the ratio of their points scored in the tournament, find how much money each team got.

9. (a) Given that $P = \begin{pmatrix} 2 & ^-1 \\ 3 & -2 \end{pmatrix}$, $Q = \begin{pmatrix} 1 & 5 \\ 2 & -3 \end{pmatrix}$ and $R = \begin{pmatrix} 4 & 3 \\ 1 & -2 \end{pmatrix}$, find

(i) QR - P (ii) the determinant of QR - P

(b) If $\begin{pmatrix} 4 & 1 \\ x & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$, determine the values of x and y.

10. Three men John, peter and David went for shopping. John bought 2 *kg* of sugar, 4 *kg* of tealeaves and one loaf of bread, peter bought 10 *kg* of sugar, 3 *kg* of tealeaves and 2 loaves of bread; David bought 5 kg of sugar, 5 *kg* of tea leaves and 3 loaves of bread. The prices are; sugar shs. 2000 per *kg*, tealeaves shs. 1000 per *kg* and shs. 1800 for one loaf of bread. Use a suitable matrix multiplication to find the total amount of money spent by each of John, peter and David.

11. (a) Given that $P = \begin{pmatrix} 2 & ^-1 \\ 4 & 3 \end{pmatrix}$, $Q = \begin{pmatrix} 1 & ^-2 \\ -2 & 5 \end{pmatrix}$ and that R = PQ, find the inverse of R.

(b) Three men, Ali, Ben and Charles went to a supermarket to buy groceries. Ali bought 4 *kg* of rice, 2 *kg* of sugar and 3 *kg* of Posho. Ben bought 4 *kg* of Posho and 3 *kg* of Sugar. Charles bought 6 *kg* of sugar, 2 *kg* of rice and 3 *kg* of Posho

(i) Write down a 3 × 3 matrix for the purchases.

(ii) If a *kg* of rice costs shs.2500, a *kg* of sugar shs3,000 and a *kg* of Posho shs 2,000, write down a 3 × 1 matrix for the costs.

(c) By matrix multiplication, determine the amount of money paid by each customer

(d) Find the supermarket's total earning from the three customers.

12. Three ladies; Peace, Queen and Ritah went to a certain supermarket. Peace bought 2 *kg* of sugar, 1 *kg* of tea leaves and 2 leaves of bread. Queen bought 10 *kg* of sugar and 2 loaves of bread while Ritah bought 5 *kg* of sugar, 2 *kg* of tea leaves ad 1 loaf of bread. The cost of a kilogram of sugar was shs 1800, a kilogram of tea leaves was shs 900 and a loaf of bread shs 1400.

(i) Write down a 3 × 3 matrix for the items bought.

(ii) Write down a 3 × 1 cost matrix.

(iii) By matrix multiplication, determine the total expenditure by the three ladies.

13. (a) Given that matrix $A = \begin{pmatrix} 1 & 2 \\ 4 & 5 \end{pmatrix}$, $B = \begin{pmatrix} ^-1 & 1 \\ 3 & 2 \end{pmatrix}$ and $c = \begin{pmatrix} 4 & 6 \\ 10 & 15 \end{pmatrix}$, find matrix N such that A2 +3B − C.

(b) Use the matrix method to solve the simultaneous equations. $a - 2b = 3$

$$2a = 11 - b$$

14. (a) Given the matrices; $A = \begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 1 & ^-2 \\ 3 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix}$. Find a matrix M such that $M = 2AB + 3C^2$.

(b) A boarding school uses 15 bags of maize, 8 bags of beans, 16 bags of maize flour, and 4 bags of rice in first term. The prices are shs. 1000, shs 1200, shs 1400 and shs 1400 respectively. In second term, the school uses 16 bags of maize flour and 5 bags of rice at shs.1400, shs.2600, shs.1600 and shs.1500 respectively. In third term, the school uses 12 bags of maize, 5 bags of beans, 12 bags of maize flour and 3 bags of rice at shs.1800, shs. 2200, shs. 2000, and shs. 1500 respectively. Use matrices to find the total cost of food stuff that year.

15. (a) If $B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ and $A = \begin{pmatrix} ^-2 & 1 \\ 1 & 0 \end{pmatrix}$, find; (i) AB (ii) $(AB)^{-1}$

16. Alpha made an order of the items in Pallisa supermarket as follows;

	½ kg packet	1 kg packet	2 kg packet	5 kg packet
Kaboloi Millet	0	4	2	0
Agole millet	4	2	5	2
Kumi millet	0	3	1	4

The table below shows the cost of each size of the packet.

½ *kg* packet	800
1 *kg* packet	1500
2 *kg* packet	2800
5 *kg* packet	6500

(i) Find the number of packets ordered

(ii) Write down 4 by 3 matrix for the order

(iii) Write down 1 by 4 matrix for the costs

(iv) Given that he had to pay 10% discount for the order made of the millet, use matrices to find the total expenditure by Alpha.

17. Find the values of x given that the matrix $\begin{pmatrix} x & 3x+4 \\ 1 & x+3 \end{pmatrix}$ is a singular matrix.

18. Shell petroleum company owns two pump stations, one in Mukono town and the other in Seeta Town. Over two consecutive days, the sales were recorded as follows (in litres).

First day;

Town	Petrol	Diesel	Kerosene
Mukono	1300	850	750
Seeta	1200	950	700

Second day;

Town	Petrol	Diesel	Kerosene
Mukono	1350	925	650
Seeta	1250	850	850

(a) (i) Represent the above information on 2 × 3 matrices showing the sales of each day.

(ii) Write down a matrix representing the total sales over the two days.

(b) The cost of fuel at each station of given by; Petrol shs 2300 per litre, Diesel shs 2100 per litre and Kerosene shs 1800 per litre.

(i) State a 3 × 1 cost matrix.

(ii) Using a suitable matrix multiplication, find the total income from each station.

(iii) Find the total income from both stations for the two days.

19. (a) If $A = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 1 & -2 \end{pmatrix}$ are matrices, find;

(i) AB (ii) det (AB)

(b) If $3\begin{pmatrix} 4 & 0 \\ 5 & 7 \end{pmatrix} + \begin{pmatrix} 4 & 1 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, find the values of a, b, c and d.

20. (a) Using matrix method find the values of x and y which satisfy the equations:

$$2x - 3y = 12$$
$$x + 2y + 1 = 0$$

(b) Given that matrix $A = \begin{pmatrix} 3 & -2 \\ -4 & 5 \end{pmatrix}$, find a matrix B such that $AB = \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$. Hence or otherwise find the inverse of A.

21. A supplier of food stuffs to school per week summarised his information for supplies made to Kibuli SS, Seeta HS and Buddo SS for 2 weeks as follows:

First week:

Kibuli SS: 2bags of posho,1 bag of rice and 3bags of potatoes

Seeta HS: 2bags of posho and 2 bags of rice

Buddo SS: 1 bag of posho, 1bag of rice and 2bags of potatoes.

Second week:

Kibuli SS: 3 bags of posho and 2 bags of potatoes

Seeta HS: 1 bag of posho, 2bags of rice and 1 bag of potatoes

Buddo SS: 3bags of posho and 1 bag of potatoes.

The cost of posho, rice and potatoes is shs 20,000, shs 30,000 and shs 10,000 per bag respectively.

(a) Write down 3×3 matrices for the supplies made per week for two weeks.

(b) Write down a 3×1 cost matrix

(c) Which school spent most in the two weeks?

22. (a) Solve the following simultaneous equations using matrix method, $2x - y = {}^-4$, $x + 5y = 9$

(b) Given that $(x \quad 1 - x)\begin{pmatrix} 2 & 18 \\ 2 & 6 \end{pmatrix} = 4(1 - x \quad y)$, find the values of x and y.

(c) Find x and y if $\begin{pmatrix} 3 & {}^-1 \\ 20 & y \end{pmatrix}\begin{pmatrix} x \\ {}^-1 \end{pmatrix} = \begin{pmatrix} 7 \\ 20 \end{pmatrix}$

(d) Use matrix method to solve the pair of simultaneous equations; $\begin{matrix} m + 2n = 2 \\ 4n + 1 = 2m \end{matrix}$

(e) Given that $\begin{pmatrix} x - 1 & x + 1 \\ 3x & x \end{pmatrix}$ is a singular matrix. Find the possible values of x.

23. Students of S.1 to S.3 in Rock High school were distributed in 3 streams; South, North and East as follows: S.1; 65 in South, 60 in North and 70 in East. S.2; 50 in South, 62 in North and 54 in East while S.3 had 54 in South, 60 in North and 42 in East. Each student per steam had to pay subscription for club membership as shown in the table below.

	South	North	East
Subscription fee	5,000	4,000	3,000

(a) Write down (i) a 3×3 matrix for the student population in the school.

(ii) a 1×3 matrix for subscription fees.

(b) By multiplying the two matrices in part (a) above, calculate how much money is collected per class.

(c) How much money will go for charity work if 12.5% of the S.2 subscription fee is allocated for charity work?

24. (a) Given that $\begin{pmatrix} 3 - a & 3 \\ -1 & -2 \end{pmatrix}\begin{pmatrix} {}^-3 \\ x \end{pmatrix} = \begin{pmatrix} {}^-3 \\ x \end{pmatrix}$. find the values of a and x.

(b) Given that $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$. Show that $A^2 - 4A = I$ where I is a 2×2 identity.

25. (a) Given that matrix $A = \begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix}$, find the values of the scalar k for which $A - kI$ is singular.

(b) If $\begin{pmatrix} 0 & 4 \\ 3 & -1 \end{pmatrix}$ is pre multiplied by the column matrix $\begin{pmatrix} x \\ y \end{pmatrix}$ to give $\begin{pmatrix} -8 \\ x \end{pmatrix}$, find the values of x and y.

26. Find the possible values x can take on given $A = \begin{pmatrix} x^2 & 3 \\ 1 & 3x \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 6 \\ 2 & x \end{pmatrix}$ and AB = BA.

27. Given that $D = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ and I is a 2×2 identity matrix , obtain the values of p and q such that $D^2 = pD + qI$.

28. (a) Given that matrix $P = \begin{pmatrix} 4 & 3 \\ -2 & 1 \end{pmatrix}$ and $Q = \begin{pmatrix} 11 \\ 7 \end{pmatrix}$. If PR = Q, determine:

(i) the order of matrix R. (ii) matrix R.

(b) Given that $\det\begin{pmatrix} x + 2 & 2 \\ y - 1 & 3 \end{pmatrix} = 4$ and $\det\begin{pmatrix} 2 & x + 2 \\ 3 & 1 - y \end{pmatrix} = 0$. Determine the values of x and y, hence the two matrices.

29. Given that matrix $Q = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$, find the values of k such that $Q^2 - 5Q + kI = 0$.

Answers

1. (i) $\begin{pmatrix} 3 & 2 \\ 3 & 1 \end{pmatrix}$ (ii) $^-3$

2. $x = 1, y = \ 1$

3. (a) $x = 3, y = {}^-2$ (b) $x = {}^-3, y = {}^-7$ (c) $x = 2, y = 3$

4. (a) $\begin{pmatrix} 1 & 8 \\ 0 & 9 \end{pmatrix}$ (b) $\begin{pmatrix} \frac{1}{25} & \frac{3}{25} \\ \frac{6}{25} & \frac{^-7}{25} \end{pmatrix}$

5. $a = 2, b = {}^-2, c = 2, d = 16$

6. (c) $T = \begin{array}{c} \begin{matrix} A & B & C & D \end{matrix} \\ \begin{pmatrix} 12 & 6 & 0 & 3 \\ 6 & 0 & 3 & 9 \\ 9 & 3 & 15 & 3 \\ 6 & 3 & 3 & 9 \end{pmatrix} \end{array}$ (d) $\begin{array}{c} \begin{matrix} A & B & C & D \end{matrix} \\ \begin{pmatrix} 31{,}800 & 11{,}400 & 21{,}600 & 25{,}200 \end{pmatrix} \end{array}$

7. $\begin{pmatrix} \frac{10}{7} & \frac{27}{7} \\ \frac{34}{7} & \frac{33}{7} \end{pmatrix}$

8. (a)(ii) $\begin{array}{c} \begin{matrix} W & D & L \end{matrix} \\ \begin{pmatrix} 5 & 3 & 4 \\ 3 & 3 & 6 \\ 7 & 3 & 2 \\ 4 & 3 & 5 \end{pmatrix} \end{array}$ (b) Winner : Mvara
Points : 24 (d) $\begin{pmatrix} \text{Kibuli : shs 540,000} \\ \text{Lubiri : shs 360,000} \\ \text{Mvara : shs 720,000} \\ \text{Nagalama : shs 450,000} \end{pmatrix}$

9. (a)(i) $\begin{pmatrix} 7 & ^-6 \\ 2 & 14 \end{pmatrix}$ (ii) 110 (b) $x = 2, x = {}^-6$
$y = -4$ or $y = 28$

10. $\begin{array}{l} \text{John} \\ \text{Peter} \\ \text{David} \end{array} \begin{pmatrix} 9800 \\ 26{,}600 \\ 20{,}400 \end{pmatrix}$

11. (a) $R = \begin{pmatrix} 4 & ^-9 \\ ^-2 & 7 \end{pmatrix}$ Inverse of $R = \frac{1}{10}\begin{pmatrix} 7 & 9 \\ 2 & 4 \end{pmatrix}$ (c) $\begin{array}{l} \text{Ali} \\ \text{Ben} \\ \text{Charles} \end{array} \begin{pmatrix} 22{,}000 \\ 17{,}000 \\ 29{,}000 \end{pmatrix}$ (d) shs.68,000

12. (iii) shs 40,300

13. (a) $N = \begin{pmatrix} 2 & 9 \\ 23 & 24 \end{pmatrix}$ (b) $a = 5$
$b = 1$

14. $M = \begin{pmatrix} 22 & -6 \\ 9 & 23 \end{pmatrix}$ (b) shs 136,300

15. (i) $AB = \begin{pmatrix} ^-2 & ^-3 \\ 1 & 2 \end{pmatrix}$ (ii) $\begin{pmatrix} ^-2 & ^-3 \\ 1 & 2 \end{pmatrix}$

16. (iv) shs 70,290 (17) $x = \pm 2$

18.(a)(ii) $\begin{pmatrix} 2650 & 1775 & 1400 \\ 2450 & 1800 & 1550 \end{pmatrix}$ (b)(ii) $\begin{array}{l} \text{Mukono} \\ \text{Seeta} \end{array} \begin{pmatrix} 12{,}342{,}500 \\ 12{,}205{,}000 \end{pmatrix}$ (iii) shs 24,547,500

19. (a)(i) $AB = \begin{pmatrix} 5 & ^-4 \\ 5 & ^-8 \end{pmatrix}$ (ii) $^-20$ (b) $a = 21 \quad c = 19$
$b = 16 \quad d = 22$

20.(a) $x = 3$
$y = {}^-2$ (b) $B = \begin{pmatrix} 5 & 2 \\ 4 & 3 \end{pmatrix}$ Inverse of $A = \frac{1}{7}\begin{pmatrix} 5 & 2 \\ 4 & 3 \end{pmatrix}$

21.(c) Seeta = 190,000/=

22.(a) $x = {}^-1$
$y = 2$ (b) $x = \frac{1}{2}$
$y = 20$ (c) $x = 2$
$y = 20$ (d) $m = 1.25, n = 0.375$ (e) $x = 0$ or $x = {}^-2$

23.(b) $\begin{array}{l} \text{S.1} \\ \text{S.2} \\ \text{S.3} \end{array} \begin{bmatrix} 775{,}000 \\ 660{,}000 \\ 636{,}000 \end{bmatrix}$ (c) shs 82,500 (24) $a = 1, x = 1$

25.(a) $k = 4$ or $k = {}^-1$ (b) $x = {}^-1, y = {}^-2$

26. $x = 3, x = \frac{^-1}{2}$ (27) $(p = 2, q = {}^-1)$

28. (a)(i) 2 x 1 matrix (ii) $R = \begin{pmatrix} ^-1 \\ 5 \end{pmatrix}$ (b) $x =\ , y = 0$ (29) $k = 6$

SURDS, INDICIES AND LOGARITHM

TRIAL QUESTIONS

1. Simplify ; $\frac{12m^2 \times 2m}{3mn}$ **[Ans: $\frac{8\ m^2}{n}$]**
2. Without using tables or calculator, find the value of x: $Log(3x + 8) - 3 \log 2 = log(x - 4)$ **[Ans: $x = 8$]**
3. Without using calculators and math tables, evaluate $\frac{64 \times 4^5 \times 128}{2^4 \times 32^3}$ **[Ans: 16]**
4. Without using calculators or tables simplify $\frac{1}{2}log_{10}16 + 2log_{10}\left(\frac{a}{5}\right) + log_{10}a^2$ **$\left[\text{Ans: } 2log_{10}\left(\frac{2a^2}{5}\right)\right]$**
5. (a) Use math tables to find the log of 0.0234 **[Ans: $\bar{2}.3692$ or $^-1.6308$]**
   (b) What is the antilog of 4.098? **[Ans: 12530]**
   (c) Evaluate without using tables or calculators:
   (i) $log16 - log96 + log6000$ **[Ans: 3]** (ii). $log27^{\frac{1}{3}}$ **[Ans: *log*3]**
6. Simplify $\frac{3^3 \times 9^2 \times 125}{93}$ **[Ans: 375]**
7. Given that $16^x = \left(\frac{1}{2}\right)^{x-5}$ find the value of x. **[Ans: $x = 1$]**
8. Find the value of a , if $Log_a 2 + log_a 4 = 3$ **[Ans: $a = 2$]**
9. Without using tables and calculators find the value of $\text{Log}_{10}4000 + \log_{10}500 - \log_{10}20$ **[Ans: 5]**
10. (a) Without using mathematical tables or calculator simplify $\left[\frac{27}{8}\right]^{\frac{-2}{3}} \times \left[\frac{1}{81}\right]^{\frac{1}{4}}$ **[Ans: $\frac{4}{27}$]**
    (b) Evaluate using logarithms to find the cube root of 0.0271 correct to three decimal places. [**Ans**: 0.300]
    (c) Solve $[8]^{(x+1)} = [2]^{(x+9)}$ **[Ans: $x = 3$]**
11. (a) Find x if: $81^x = \left[\frac{1}{3}\right]^{x-5}$ **[Ans: $x = 1$]**
12. Without using tables or calculators work out $\left(\frac{27}{8}\right)^{\frac{2}{3}} \times \left(\frac{16}{81}\right)^{-\frac{1}{2}}$ **[Ans: $\frac{81}{16}$]**
13. Given that $\frac{3}{2}loga^3 - loga^{\frac{1}{2}} - 2loga = 4$, find a **[Ans: $a = 100$]**
14. Express $\frac{13}{3\sqrt{2} - \sqrt{5}}$ in the form $a\sqrt{m} + b\sqrt{n}$ where a,b n and m are Integers **[Ans: $3\sqrt{2} + \sqrt{5}$]**
15. (a) Solve for x in the equation $3^x = \frac{1}{81}$ **[Ans: $x = {}^-4$]**
    (b) Use *log* tables to find the value of $\frac{218 \times 0.024}{95.43}$ **[Ans: 0.0548]**
    (c) If $\frac{\sqrt{2}}{\sqrt{3} + \sqrt{2}} = a + \sqrt{b}$, find the values of a and b **[Ans: $^-2 + \sqrt{6}$, $a = {}^-2$, $b = 6$]**
    (d) Evaluate without using tables or calculators $\frac{4}{5}log32 + log50 - 3log2$ **[Ans: 2]**
16. Express $\frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$ in the form $a + b\sqrt{c}$ where a, b and c are integers. Hence state the values of a, b and c.
    [Ans: $5 + 2\sqrt{6}$, $a = 5$, $b = 2$, $c = 6$]
17. Express $\frac{3}{3\sqrt{2} - 2\sqrt{5}} - \frac{1}{2\sqrt{2} + 2\sqrt{5}}$ in the form $a\sqrt{m} + b\sqrt{n}$ where a, b, m and n are integers. **[Ans: $\frac{19}{6}\sqrt{5} + \frac{26}{6}\sqrt{2}$]**
18. (a) Without using tables or calculator find the value of x if $log_5(x-2) + log_5(2x) - log_5(6x) = 1$ **[Ans: $x = 17$]**
    (b) Solve for x in $3^x \div \frac{1}{81^{(1-x)}} = 1$ **[Ans: $x = \frac{4}{3}$]**

19. If $\frac{4\sqrt{3}}{\sqrt{5}-\sqrt{3}} = a + b\sqrt{c}$. Find the values of a, b and c. **[Ans: $6 + 2\sqrt{5}$, $a = 6$, $b = 2$, $c = 5$]**

20. (a) If $\frac{\sqrt{5}-1}{2\sqrt{5}-4} = a + b\sqrt{5}$, find the values of a and b where a and b are constants.
[Ans: $\frac{3}{2} + \frac{1}{2}\sqrt{5}$, $a = \frac{3}{2}$, $b = \frac{1}{2}$, $c = 5$]

(b) Find the value of x, if, $log(2x - 11) - log2 = log3 - logx$. **[Ans: $x = 6$]**

(c) If $x^3 = 3.375$, use tables to find the value of x, correct to three significant figures. **[Ans: 1.50]**

(d) Given that $log_{10}2 = 0.3010$ and $log_{10}3 = 0.4771$, find without using tables or calculator, the value of;

(i) $log_{10}72$ **[Ans: 1.8572]**

21. (a) Given that $log_{10}2 = 0.3010$, $log_{10}3 = 0.4771$ and $log_{10}5 = 0.6990$, evaluate $log_{10}150$ [**Ans**: 2.1761]

(b) Use logarithms tables to evaluate; $\sqrt[3]{\frac{0.3215 \times 1.439}{0.00485}}$ **[Ans: 4.568]**

22. (a) Without using mathematical tables or calculator, simplify, $\frac{64^{-\frac{1}{3}}}{27^{-\frac{1}{3}}}$ **[Ans: $\frac{3}{4}$]**

b) Given that; $\sqrt{15} = 3.873$, $\sqrt{6} = 2.450$ and $\sqrt{10} = 3.162$, without using a calculator, evaluate to two significant figures; $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ **[Ans: 0.387]**

23. (a) Without using tables or calculators, solve the equation; $log(10x + 5) - log(x - 4) = log100$ **[Ans: 4.5]**

(b) Use logarithims to find the square root of 0.0576. **[Ans: 0.2400]**

(c) Given that $8^{2n} \times 3^m = 36$, find the values of n and m. **[Ans: $n = \frac{1}{3}$, $m = 2$]**

(d) Using mathematical tables, evaluate; $\frac{87.5 \times 0.0243}{0.003142}$ **[Ans: 676.7]**

24. (a) Solve for x given that $log_{125}x + log_{125}5x = 1$. **[Ans: $x = 5$]**

(b) If $\frac{2+\sqrt{3}}{2-\sqrt{3}} = pq\sqrt{r}$, state the values p, q and r. **[Ans: $p = 7$, $q = 4$, $r = 3$]**

25. (a) Without using tables or calculator, find the value of $2log_{10}50 + log_{10}80 - log_{10}2$

(b) (i) Express 150 as a product of its prime factors.

(ii) Using your result in (i) above, find $log_{10}150$ given that $log_2 2 = 0.3010$, $log_{10}3 = 0.4771$ and $log_{10}5 = 0.6990$. **[Refer to number 21 above]**

(c) Without using tables or calculator, evaluate $4log_{10}2 - log_{10}48 + log_{10}30$. **[Ans: 1]**

(d) Solve the equation ; $2logx = log(x + 6)$ **[Ans: $x = 3$ and $x = {}^-2$]**

26. (a) Express $\sqrt{20} + \sqrt{125} - \sqrt{45}$ in the form $a\sqrt{b}$ where a and b are constants. **[Ans: $4\sqrt{5}$]**

(b) Express $\frac{2}{2-\sqrt{3}}$ in the form $a + b\sqrt{c}$, where a, b and c are constants. **[Ans: $4 + 2\sqrt{3}$]**

27. (a) Without using tables or calculator, evaluate $2log_{10}5 + log_{10}80 - log_{10}20$. **[Ans: 2]**

(b) Use the fact that $log_{10}3 = 0.4771$ and $log_{10}5 = 0.6990$ to evaluate $log_{10}3375$. **[Ans: 3.5283]**

28. (a) Solve for x in the equation $2^{(3x-1)} \times 8^{(x-1)} = 256$. **[Ans: $x = 2$]**

(b) Given that $log_{10}2 = 0.3010$, $log_{10}3 = 0.4771$ and $log_{10}5 = 0.6990$ find the value of $log_{10}4320$ **[Ans: 3.6353]**

(c) With out using tables or calculators simplify $\left[\frac{8}{125}\right]^{\frac{-2}{3}} \left[\frac{5}{8^{\frac{-1}{2}}}\right]^{-2}$ **[Ans: $\frac{1}{32}$]**

(d) Simplify, $\frac{4^2 \times 2^2 \times 16^{\frac{1}{2}}}{8^3}$ **[Ans: 1]**

29. (a Use log table to evaluate; $\sqrt[5]{\frac{3.15 \times 0.047}{9.24 \times 26.1}}$ **[Ans: 0.2278]**

(b) Find the value of x in $log(x + 3) = log(5 - x)$. **[Ans: $x = 1$]**

(c) Without using tables or a calculator, evaluate $4log2 - log48 + log30$ **[Ans: 1]**

(d) Without using tables or calculator, evaluate $\frac{5^{-6} \times 27^4 \times 15^8}{81^5}$ **[Ans: 25]**

(e) Use tables to evaluate; $(0.0367)^3 \times 47.9$ **[Ans: 0.002368]**

(f) Simplify: $\frac{2^x \times 8^{(x-1)}}{16^{(x-1)}}$. **[Ans: 2]**

30. (a) Simplify; $\frac{24^2 \times 27^3}{2^6 \times 81}$ (b) Given that $128^a = 16^b$, find $\frac{a}{b}$. (c) Solve for x in $\frac{x^2}{7} = \frac{49}{x}$
[Ans: 2187] **[Ans: $\frac{a}{b} = \frac{4}{7}$]** **[Ans: $x = 7$]**

31. (a) Without using mathematical tables, solve for x in the equation; $9^{\left(2x-\frac{1}{4}\right)} \times 27^{\left(x-\frac{1}{2}\right)} = 729^{\left(x+\frac{1}{3}\right)}$ **[Ans: $x = 4$]**
(b) Given that $9^x = 243$, find the value of x. **[Ans: $x = 2.5$]**
(c) Evaluate without using tables or a calculator $log_{10}120 - log_{10}36 + log_{10}3$. **[Ans: 1]**

32. (a) Solve the equation for x; $\sqrt[3]{\left[\frac{2}{5}\right]^{2-x}} = \frac{8}{125}$ **[Ans: $x = ^-7$]**
(b) Evaluate without using tables or calculator $4log_{10}2 + log_{10}5 - 3log_{10}2$. **[Ans: 1]**
(c) Given that $log_{10}2 = 0.3010$ and $log_{10}3 = 0.4771$evaluate without using tables or a calculator $log_{10}0.6$.
[Ans: $^-0.2219$]

33. (a) Evaluate; $\left[\frac{27}{125}\right] \times \left[\frac{243}{3125}\right]^{\frac{-2}{5}}$ **[Ans: $\frac{3}{5}$]**
(b) Given that $\frac{6}{2\sqrt{5} + \sqrt{2}} = a\sqrt{5} + b\sqrt{2}$, determine the values of a and b. **[Ans: $a = \frac{2}{3}$, $b = \frac{^-1}{3}$]**
(c) Express $\frac{\sqrt{3} - 2\sqrt{2}}{\sqrt{3} + \sqrt{2}}$ in the form $a + b\sqrt{c}$, where a, b and c are real numbers. **[Ans: $7 - 3\sqrt{6}$]**
(e) Solve the equation $\log_{10}(7x + 2) - log_{10}(x - 1) = 1$. **[Ans: $x = 4$]**

34. (a) It is given that $log_{10}3 = 0.4771$ and $log_{10}2 = 0.3010$. Find the value of $log_{10}\left[\frac{108}{64}\right]$. Without using a calculator or tables. **[Ans: 0.2273]**
(b) Solve the following equations simultaneously
$log(x - 1) + logy = 2log2$
$logx + logy = log6$ **[Ans: $x = 3, y = 2$]**
(c) Without using tables or a calculator, simplify $\frac{1}{2}log64 + 2log3 - log12$ **[Ans: $2log2$]**

35. (a) Solve for x in the equation; $3^{2x+1} = 243$ **[Ans: $x = 2$]**
(b) Find the value of x in the equation. (i) $log_x625 = 4$ **[Ans: $x = 4$]** (ii) $log_3729 = x$ **[Ans: $x = 6$]**

36. (a) Solve for x in the equation; $32^x \times 4^{(x+3)} = 2^{27}$ **[Ans: $x = 3$]**
(b) If $log_{10}5 = 0.6990$ and $log_{10}2 = 0.3010$, find the value of $log_{10}500$ **[Ans: 2.699]**
(c) Use logarithms tables to evaluate; $\frac{685.3}{29.86}$. **[Ans: 22.95]**
(d) Simplify; $x^2 + (\sqrt{3})x$ if $x = (2 - \sqrt{3})$. **[Ans: $2(2 - \sqrt{3})$]**

37. (a) Simplify; $\frac{2^{-2} \times 3^6}{2^{-4} \times 9 \times 54}$. **[Ans: 6]**
(b) Evaluate without using tables or calculators; $2log_{10}6 + 2log_{10}5 - 2log_{10}3$ **[Ans: 2]**
(d) Use tables to evaluate; $\sqrt[3]{18.95 \times 4.2}$ **[Ans: 4.301]**

38. (a) Given that; $\frac{4\sqrt{5} + 3\sqrt{2}}{2\sqrt{2} + \sqrt{5}} = p + q\sqrt{r}$, find the values of p, q and r. **[Ans: $p = \frac{^-8}{3}$, $q = \frac{5}{3}$, $r = 10$]**
(b) Use logarithm tables to evaluate; $(1.58 \times 37.25)^{\frac{1}{3}}$ **[Ans: 3.890]**
(c) Find the value of x in the equation $2log_{10}x + log_{10}6 - log_{10}15 = 1$. **[Ans: $x = 5$]**
d) Given that $log_{10}a = 1.699$ and $log_{10}b = 1.913$, evaluate $log_{10}a^2b^{\frac{1}{3}}$ **[Ans: 9.137]**
(e) Use logarithms tables to evaluate; $\sqrt[4]{\frac{4.562 \times 0.38}{0.82}}$ **[Ans: 1.206]**

39. (a) Find the value of y if $\frac{243 \times 3^{2y}}{729 \times 3^y \times 3^{(2y-1)}} = 81$ **[Ans: $y = ^-4$]**
(b) Without using tables or calculators, evaluate the following
(i) $3log_{10}2 + log_{10}200 - log_{10}1.6$ **[Ans: 3]** (ii) $32^m \times \frac{1}{8} \times 4^{(m-3)} = 2^{12}$ **[Ans: $m = 3$]**
(iii) Using mathematical tables evaluate: $\frac{35.6}{1.47 \times 12.6}$ **[Ans: 1.922]**

40. (a) Simplify: $\frac{1}{3 - \sqrt{2}} + \frac{1}{3 + \sqrt{2}}$ **[Ans: $\frac{6}{7}$]**
(b) Without using tables or calculator, solve the equation: $log_{10}(10x + 5) - log_{10}(x - 4) = 2$ **[Ans: $x = 4.5$]**
(c) Use logarithm tables to evaluate; $\sqrt[3]{\frac{0.3216 \times 62.58}{41.57}}$ to 3 significant figures. **[Ans: 0.785]**

41. (a) Simplify; $\frac{4m^6 \times 3m^3 \times 2m^2}{8m^2 \times 3m^7}$ **[Ans: m^2]**

(b) Simplify the following;

(i) $log_5 x - log_5 2x = 2$ **[Ans: $x = 50$]** (ii) $log_{10} 4a + log_{10} 3a$ **[Ans: $2log_{10} 12a$]**

(iii) $log_2 y^2 + log_2 y$ **[Ans: $3log2y$]** (iv) $5log_{10} 2 + log_{10}\left[\frac{1}{32}\right]$ **[Ans: 0]**

(v) $log_3 3^{\frac{1}{2}} + \frac{1}{2}log_3 3$ **[Ans: 1]** (vi) $log_4 x^2 + log_4 x^3 + log_4 x^4$ **[Ans: $9log_4 x$]**

(vii) $log_{10} z^{10} - log_{10} z^7 + log_{10} z^6$ **[Ans: $9log_{10} z$]** (viii) $3log_8 x + log_8 x^3 - log_8 x^{\frac{1}{2}}$ **[Ans: $\frac{11}{2}log_8 x$]**

(c) Without using tables or calculators evaluate; $\sqrt[3]{0.002406}$ to 3*dp*. **[Ans: 0.134]**

(d) Given that $log_{125} x + log_{125} 5x = \frac{1}{3}$, find the values of x. **[Ans: $x = \pm 1$]**

(e) Given that $log_{10} 7 = 0.845$ and $log_{10} 2 = 0.301$, find $log_{10}\left[\frac{49}{64}\right]$ **[Ans: $\bar{}0.116$]**

42. Use logarithm tables to evaluate the following;

(a) $\sqrt[3]{72.4 \times 0.569}$ **[Ans: 3.45]**

(b) 8.05×23.5^2 **[Ans: 4450]**

(c) $19.7 \div (8.27)^2$ **[Ans: 0.288]**

(d) $\frac{3.74 \times 0.582}{0.0507}$ **[Ans: 42.9]**

(e) $3\sqrt{11} \times 5.132$ **[Ans: 58.5]**

(f) $\sqrt[3]{3.94 \times 0.632}$ **[Ans: 1.36]**

(g) $\frac{0.078 \times 34.3}{4.8}$ **[Ans: 0.557]**

43. The formula for the radius of a circle is $r = \frac{A}{\pi}$, when A is its area. Using logarithm tables, find r when A = 16.9 and $\pi = 3.14$. **[Ans: 2.32]**

KINEMATICS

RECALL

Remember the following key issues before attempting any question

◊ All drawings in kinematics must be drawn on a graph paper

◊ Be in position to convert from one metric unit to another. Eg km to m or cm etc.

◊ Neatness is a must requirement that earns you a bonus mark.

◊ You must know how to calculate speed, distance and time.

◊ The formula for distance = speed × time = S × T. This formula has to be mastered.

◊ Read the question at least two times before starting to attempt any kinematics numbers.

TRIAL QUESTIONS

1. Sylvester left UMSS at Noon and cycled towards Jinja 100 *km* away at a steady speed of 16 kmh^{-1}. After cycling for 180 minutes he rested for one hour at Lugazi. Then continued his journey with the same speed up to Jinja. Forty five minutes after the departure of Sylvester, Michael also started walking from UMSS towards Jinja at 6 kmh^{-1} for 1 hour and reached Kireka. He waited for 45 minutes at Kireka and later boarded a bus travelling at governed speed 32 kmh^{-1}. It travelled for one hour and stopped at Namawojolo for another one hour; then resumed its journey at 42 kmh^{-1} non stop up to Jinja.

   (a) Using a scale of 2 *cm*: 1 hour and 2 cm: 10 *km* draw their distance time graphs on the same axes.

   (b).From your graph determine;

   (i).the time and distance from UMSS the bus overtook the cyclist

   (ii).the distance between Namawojolo and Lugazi.

   (iii).How long did Michael wait before Sylvester joined him at Jinja sailing club.

2. Towns A and B are 300 *km* apart by road. At 7.00 *a.m*. Ojok leaves town A for town B driving a pick-up at a steady speed of 80 kmh^{-1}. After driving for 2 hours, the pick-up breaks down and he is forced to stop for half an hour after which he continues towards B at a reduced speed of 60 kmh^{-1} up to town B. One and a half hours after Ojok left town B, Kase leaves town B for town A driving at a constant speed non-stop all the way to A. Using a scale of 1 *cm* to 1 hour on the horizontal axis and 1 *cm* to 50 *km* on the vertical axis, draw the graph of both men's journeys on the sasme pair of axes, given that both reached their respective destinations at exactly the same time. Use your graph to find

   (a) Kase's speed in kmh^{-1}

   (b) When they met and how far this was from town A

   (c).Their time of arrival.

3. Gulu is 360 *km* away from Kampala. At 7:30 *a.m*. a taxi mini-bus leaves Gulu for Kampala travelling at a steady non stop speed of 50 kmh^{-1}. Two hours later a Gateway bus travelling at a steady non stop speed of 80 kmh^{-1} left Kampala for Gulu.

   (a) Using a scale of 2 *cm* to represent 1 hour and 2 *cm* to represent 50 *km*, draw on the same axes graphs showing the journeys of the two vehicles

   (b).Using your graphs determine the:

   (i).distance from Gulu where the two vehicles met

   (ii).time when the two vehicles met

   (iii).times when the vehicles arrive at their stations

   (iv).difference in the times of arrivals of the vehicles at their respective stations.

4. A taxi started its journey from Kampala at 0620 hours traveling at constant speed of 80 *km/hr* to Masindi 200 *km* away. After traveling for $1\frac{1}{2}$ hours it broke down, and put right for 1 hour then resumed its journey using an average speed of 30 kmh^{-1} to Masindi. At 0720 hrs the express bus left Masindi for Kampala traveling at a constant non stop speed of 50 kmh^{-1}. using a scale of 2 *cm*: 20 *km* 2 *cm*: 1 *hr*. Draw a distance time graph for the two vehicles on the same axes. From your graph find
   (i) when and where from Masindi the bus meet the taxi
   (ii) the difference in the time of arrival of the two vehicles
   (iii) the average speed of taxi for the whole journey.
5. Arua town is 540 *km* from Kampala city. A pajero left Kampala for Arua at 7:00 *am* traveling at a steady speed of 100 kmh^{-1}. After traveling for 2 hours, the pajero had a mechanical breakdown which took exactly 2 hours to repair after which it continued non-stop to Arua at a steady speed of 110 kmh^{-1}. One hour after the pajero left Kampala, a bus left Arua for Kampala traveling at a steady speed of 80 kmh-1 but had a stop of 30 minutes at Karuma which is 240 *km* from Arua town. The bus then continued non-stop to Kampala at a steady speed of 20 kmh^{-1} more than its original speed.
   (a) Using a scale of 2 *cm* to represent 1 hour on the horizontal axis and 2 *cm* to represent 50 *km* on the vertical axis, draw on the same axes a distance-time graph for the two vehicles.
   (b) Using your graphs, determine the;
   (i) time at which the two vehicles met.
   (ii) distance form Kampala at which the two vehicles met.
   (iii) time of arrival of the pajero in Arua town.
   (iv) time of arrival of the bus in Kampala city.
   (v) difference in the times of arrival of the two vehicles at their respective stations.
6. Mbale and Kampala are 165 *km* apart. A bus leaves Mbale at 8:00 *am* traveling at 40 *km*/*hr*. 45 minutes later it breaks down and stops for an hour. It resumes its journey and reaches Kampala at 11:30am. A tax leaves Kampala at 9:30 *am* an travels at a steady speed of 60 *km*/*hr* to Mbale.
   (a) On the same pair of axes draw both journeys described above.
   (b) When did the taxi reach Mbale?
   (c) What was the speed of the bus on the final leg?
   (d) When and where did the two vehicles meet?
7. Soroti is 340 *km* from Kampala. At 7:00 *am* a car traveling at a steady non-stop speed of 60 kmh^{-1} leaves Kampala for Soroti. One hour and 20 minutes later a bus leaves Kampala for Soroti traveling at a constant speed; After an hour of traveling the bus gets a puncture at a place 100 *km* from Kampala. It takes 40 minutes to fix the tyre and then resumes traveling at a steady non-stop speed so that it reaches Soroti at mid-day.
   (a) Using scales of 2 *cm* to represent 40 *km* on the vertical axis and 3 *cm* to represent 1 hour on the horizontal axis, draw on the same axes graphs of the journeys of the two vehicles.
   (b) Using your graphs, estimate the;
   (i) time at which the bus overtakes the car
   (ii) distance from Soroti where the bus overtakes the car
   (iii) time when the car arrives in Soroti
   (iv) time taken by the bus to travel from the point of fixing the tyres to Soroti
   (v) time taken by the bus driver to wait for the driver of the car in Soroti.
8. Towns A and B are 45 *km* apart. James starts from A at 8:00 *am* and travels towards B at 20 *km*/*hr*. When he is 15 *km* from A, he changes speed so that he arrives at B at 10:30am. David leaves town B at 8:30 *am* and travels to A at a speed of 28.4 *km*/*hr*. Taking 2 *cm* to represent 5 *km* on the vertical axis and 1 *cm* on to represent 15 minutes on the horizontal axis, draw graphs on the same axes to find,
   (i) the time James and David met
   (ii) the speed of James in the last part of the journey

(iii) the time when David reaches A.

9. A car travelling at a steady speed of 50 *km*/*hr* leaves town A at 8:00 *am* for town B, 180 *km* away. Two hours later it breaks down. It is repaired for $1\frac{1}{2}$ hours before resuming the journey at the same speed. At 10:00 *am* a bus leaves town B for town A non-stop at a steady speed of 40 *km*/*hr*.
   (a) Using a scale of 2 *cm* representing 1 hour on the time axis and 2 *cm* representing 20 *km* on the distance axis, draw distance – time graphs of the two vehicles on the same axes.
   (b) Use your graphs to determine;
      (i) When and where from town B the two vehicles bypass each other.
      (ii) The times of arrival of the two vehicles at their respective destinations.

10. Charles moved from town A to town B, 31.5 *km* away at a speed of 10.5 kmh^{-1}. He covered the remaining distance to town C at a speed of 6 kmh^{-1}. If the total time taken was $5\frac{1}{4}$ hours,
   (a) find the distance between B and C.
   (b) the average speed for the journey

11. Peter starts cycling from his home at exactly 8 *am*, at a steady sped of 16 kmh^{-1} to attend a meeting at the Gombolola headquarters, 20 *km* away. At the same time, Opela is sent from the Gombolola headquarters and comes at a steady speed of 6 $kmhr^{-1}$ to inform Peter that the meeting was postponed. Unfortunately, at exactly 8:30 *am*, Peter gets a puncture and works on it for 20 minutes before proceeding his journey, cycling carefully at 8 kmh^{-1}. Using a scale of 1cm to 1 *km* on vertical axis and 1 *cm* to 10 minutes on horizontal axis.
   (a) draw the distance – time graph on the same axes for both Peter and Opela.
   (b) Use your graphs to determine;
      (i) the time Peter met Opela
      (ii) the time when they were 3 *km* apart
      (iii) the distance from the point where they met to the Gombolola headquarters.

12. Two towns, S and T, are connected by a road. The distance between S and T is 11 *km*. John leaves town S at 10:00 hours and cycles along this road at a constant speed of 12 *km*/*h*. After 30 minutes he stops and rests for 10 minutes. He then continues his journey to T at a constant speed, arriving at 11:00 hours.
   (a) (i) Draw the distance – time graph for John's journey, using a scale of 2 *cm* : 10 minutes for the horizontal axis and 2 *cm* : 1 unit for the vertical axis.
      (ii) At what speed does John cycle after the rest.

   Michael leaves town T at 10 00 hours. He walks along the same road towards S for one hour at a constant speed of 3 *km*/*hr*.
   (b) On the same graph, draw the distance-time graph for Michael's journey.
   (c) Write down the time at which John and Michael meet and the distance from town T.

13. A man leaves his home at 8.00 *a.m* cycling to a market 21 *km* away at a speed of 20 *km*/*h*. At the same time his wife leaves the market; riding back home at a speed of 8 *km*/*hr*.
   (a) Calculate (i) how far from home they will meet each other.
      (ii) at what time they will meet.
   (b) If the man's bicycle develops a fault immediately after meeting his wife hence forcing him to reduce the speed to 8 *km*/*hr*, at what time will he reach the market?

14. A lorry set off from Tororo at 07:30 hours at a steady speed of 40 *km*/*hr* to Kampala, a distance of 180 *km* away. After travelling for 2 hours it stopped and rested for $1\frac{1}{2}$ hours, then continued at asteady speed of 50 km/hr for the rest of it's journey. A car also set off from Kampala to Tororo at the same time as the lorry at a steady speed of 60 *km*/*hr* but suddenly reduced it's speed after 2 hours to 15 *km*/*hr* due to some mechanical fault for the remaining journey. Using scales of 1 *cm* to 10 *km* and 1 *cm* to 30 minutes on the vertical and horizontal axes respectively;
   (a) Draw distance time graphs showing the routes of the two vehicles

(b) Using your graphs determine the

(c) (i) distance between the two vehicles after 2 hours

(ii) difference in time of arrival at respective.

15. A car leaves town A at 9:30 *a.m* for town B, 100 *km* away at a steady speed of 60 *km*/*hr*. After reaching town B the car rests for 20 minutes before resuming it's journey back to town A at a constant speed thus reaching A at 12:40 *p.m*.

 (a) Using a scale of 2 *cm*:30 minutes on the *x*-axis and 2 *cm*: 10 *km* on the *y*-axis, draw the graph of this motion.

 (b) Use your graph to determine

 (i) the time at which the car reached town B.

 (ii) the speed at which the car travelled back to town A.

16. Towns A and B are 300 *km* apart. At 9:30 *a.m*, Mukasa was 60 *km* away from town A moving towards town B on a Mate motor bike, when Byaruhanga set off from A on a boxer moving non-stop at 50 *km*/*hr* towards B. At 2:42 *p.m*, Byaruhanga overtook Mukasa and they continued with their journey. By using calculation, determine;

 (i) the distance from A when Byaruhanga overtook Mukasa.

 (ii) Mukasa's average speed (to 1 decimal point).

 (iii) Times when Byaruhanga and Mukasa arrived at B.

 (iv) Difference in times of arrival

17. The distance between Nakasongola and Gulu is 130 *km*. car *x* starts from Nakasongola at 8.00 *am* and travels at 40 *km*/*hr* towards Gulu. A second car y starts travelling from Gulu at 8.15 *am* towards Nakasongola at an average speed of 35 *km*/-. At what time will the two cars meet?.

18. A car leaves town A at 9.30 *am* for town B, 100 *km* away, at a steady speed of 60 *km*/*hr*. After reaching town B, the car rests for 20 minutes, before resuming it's journey back to town A at constant speed thus reaching A at 12.40 *p.m*

 (a) Using a scale of 3 *cm* : 30 min on the time -axis and 2 *cm*: 10 *km* on the Distance-axis; draw the graph for this motion.

 (b) Use your graph to determine;

 (i) the speed at which the car travelled back to A

 (ii) the time at which the car reached town B

19. A bus sets off from town A, at 7:30 *am* at a steady speed of 60 *km*/*hr* to town B, 240 *km* away. After travelling for 90 minutes it stopped at a service centre for 30 minutes after which it increased it's speed by 15 *km*/*hr* until it's final destination. 30 minutes later after the departure of the bus, a taxi sets off from town B towards town A at a steady speed and arrives 15 minutes before the bus.

 (a) Using scales of 2 *cm* to represent 30 minutes and 2 *cm* to represent 30 *km*, draw on the same axes distance – time graphs showing the journeys for the two vehicles.

 (b) Using your graph(s) in (a) above, find the time;

 (i) when the vehicles by pass each other

 (ii) taken by the taxi to cover the journey

 (c) Calculate the average speed of the journey.

20. The distance from Jinja to kampala is 73 *km*, kampala to Gayaza is 17 *km* and Mwiri to Jinja is 20 *km*. This term, S4 students from Gayaza went for a geography field work to Mwiri. The bus left Mwiri to Gayaza at 5:00 *pm* and travelled at an average speed of 60 *km*/*hr*. it stopped at Namawojolo for 15 minutes so that students can buy roasted chicken. Namawojolo is about 40km from jinja. It then proceeded non-stop to Gayaza at an average speed of 40 *km*/*hr* because of traffic jam. At 5:30 *pm* a teacher left Gayaza high shool to Jinja in his car at an average speed of 80 *km*/*hr* non-stop. Assume the teacher and the school bus used the same route.

 (a) on the same axes draw graphs showing the students and the teacher's journey.

(b) Use your graph to determine the following

(i) the time the bus arrived in Gayaza.

(ii) the time and distance from Kampala when the teacher met the students.

(iii) the difference in times of arrival of the teacher and the students.

[**use a scale of 1 *cm* for 10 *km* and 4 *cm* to represent 1 hour**]

21. A cyclist **P** leaves town **B** at 1.06 *pm* for village **A** riding non- stop at a steady speed of 15 *km*/*hr* and arrives in village **A** at 3.06 *pm*. another cyclist Q leaves village **A** at noon for town **B**. from town A cyclist **Q** rides at a steady speed of 20 *km*/*hr* for 45 minutes. He then rests for 30 minutes and then continues with a steady speed of 15 *km*/*hr* and reaches town **B** at 2.15 *pm*.

(a) Represent the motion of cyclists P and Q on a distance time graph. (**Use a scale of 1 *cm* : 15 minutes on the *x*-axis, 2 *cm* : 5 *km* on the *y*-axis**)

(b) Use your graph to find;

(i) when did the two cyclists pass each other and how far from B were they at this time?

(ii) how far apart were the two cyclists at 2:00 *pm*?

22. At 7:00 *am* a cyclist left town A for town B 120 *km* away, at an average speed of 20 *km*/*hr*. after cycling for 3 hours he rested for an hour. He continued to town B at the same speed. At 7:30 *am*, a motorist left town B for town A at an average speed of 60 *km*/*hr*. He then stayed in town A 1 hour before returning to town B. the return journey to town B took him $2\frac{1}{4}$ hours.

(a) On the same axes, using a scale of 1 *cm* to represent 10 *km* and 2 *cm* to represent 1 hour, draw a distance time graph for cyclist and motorist.

(b) From your graph, determine;

(i) the times and distance from town A the motorist by passed the cyclist on his way to and from town A.

(ii) how long had the motorist been in town B before cyclist arrived

23. The distance between A and B is 432 *km*. A lorry travelling at steady speed of 72 *km*/*hr* leaves A at 6:45 *am* for town B. $1\frac{1}{2}$ hours later, a min-bus leaves from A at a steady non – stop speed of 108 *km*/*hr* heading from town B.

(a) On the same axes, show the journeys of the two vehicles. (**use scales of 2 *cm* to represent 40 *km* and 2 *cm* to represent 1 hour**)

(b) use your graph to estimate the;

(i) time and distance from A when the min-bus overtakes the lorry

(ii) the times when the two vehicles arrive in town B

(iii) the difference in time of arrival of the two vehicles.

24. The distance from Gulu to kampala is about 380km. otada bus leaves Gulu at 7:30 *am* and travels nonstop to Kampala at 60 *km*/*hr*. at 8:50 *am* the headteacher of Gulu college leaves Kampala in his car and travels towards Gulu at steady speed of 120 *km*/*hr*.

(a) Using the same axws, draw a distance-time graph showing the journeys of both vehicles, hence find when and at what distance from Kampala they meet.

(b) If the bus increases it's speed by 10 *km*/*hr*,

(i) Calculate the time at which the bus arrived in Kampala.

(ii) Determine the difference in times of arrival of the vehicles. (**use 2 *cm*** : **50 *km* and 2 *cm*:1 hr**)

25. A bus travels from a town P to another town Q and then back to P. For the outward journey the speed is 5 *km*/*h* faster than the speed for the return journey and the return journey takes half an hour longer. If the towns are 120 *km* apart and *km*/*h* is the return speed;

(i) Show that $x^2 + 5x - 1200 = 0$

(ii) Find the value of to 3 significant figures.

(iii) Use the value of in (ii) to find the time taken to reach Q from P to the nearest minute.

QUADRATIC EQUATIONS

RECALL

◊ There are three methods of solving quadratic equations.

Factorization method;

◊ A quadratic equation is of form $ax^2 + bx + c = 0$. The left hand side of equation can be broken into two factors. A quadratic equation has two roots. The roots may be equal or unequal or one of these root may be zero. (Refer to factorization of quadratic expressions).

Example; Solve the equation $x^2 + 9x - 36 = 0$

Solution;

$x^2 + 9x - 36 = 0$

factors are $^-3$ and 12

$x^2 + 12x - 3x - 36 = 0$

$x(x + 12) - 3(x + 12) = 0$

$(x + 12)(x - 3) = 0$

either

$x + 12 = 0$

$x = {}^-12$

or

$x - 3 = 0$

$x = 3$

Completing squares method

◊ Under this method, a quadratic equation $ax^2 + bx + c = 0$ is first expressed in the form; $x^2 + \frac{b}{a} + \frac{c}{a} = 0$

◊ Then divide the cofficient of x by 2. $x^2 + \frac{b}{2a} + \frac{c}{a} = 0$

◊ We add the coffiecient of x after dividing by two. $x + \frac{b}{2a}$ and then square the sum $\left(x + \frac{b}{2a}\right)^2$

◊ Subtract the square of the coeffcient of x after dividing by two from the result.

$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}$

From that we add $\frac{c}{a}$ and equate to zero. $\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$. Generally $(x + p)^2 + q = 0$

Example: Use completing squares to solve $x^2 + 3x + 2 = 0$

Solution

$x^2 + 3x + 2 = 0$

$(x^2 + 3x) + 2 = 0$

$\left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 2 = 0$

$\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} + 2 = 0$

$\left(x + \frac{3}{2}\right)^2 = \frac{9}{4} - 2$

$\left(x + \frac{3}{2}\right)^2 = \frac{1}{4}$

take square root on both sides

$\sqrt{\left(x + \frac{3}{2}\right)} = \sqrt{\frac{1}{4}}$

$x + \frac{3}{2} = \pm\frac{1}{2}$

either

$x + \frac{3}{2} = {}^-\frac{1}{2}$

$x = \frac{^-3}{2} - \frac{^-1}{2}$

$x = \frac{^-4}{2}$

$x = {}^-2$

or

$x + \frac{3}{2} = \frac{1}{2}$

$x = \frac{1}{2} - \frac{3}{2}$

$x = \frac{^-2}{2}$

$x = {}^-1$

$x = {}^-1, x = {}^-2$

Method of quadratic formula

◊ Equationof form $ax^2 + bx + c = 0$ can be solved using the formula; $x = \frac{^-b \pm \sqrt{b^2 - 4ac}}{2a}$

◊ Whenever b is negative, then the negative on b disappears. $x = \frac{-(^-b) \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$

TRIAL QUESTIONS

Use completing squares to solve the following.

1. $x^2 - 5x + 6 = 0$ **[Ans: x = 3, x = 2]**
2. $3x^2 + 8x + 4 = 0$ **[Ans: $x = {}^-2, x = \frac{{}^-2}{3}$]**
3. $2x^2 - 5x + 2 = 0$ **[Ans: $x = \frac{1}{2}, x = 2$]**
4. $16x^2 - 8x + 1 = 0$ **[Ans: $x = \frac{1}{4}, x = \frac{1}{4}$]**
5. $x^2 + 4x - 5 = 0$ **[Ans: $x = {}^-5, x = 1$]**
6. $x^2 - 24 = 2x$ **[Ans: $x = 6, x = {}^-4$]**
7. $x^2 - 11x + 28 = 0$ **[Ans: $x = 4, x = 7$]**
8. $2x^2 - 3x - 2 = 0$ **[Ans: $x = 2, x = \frac{{}^-1}{2}$]**
9. $4x^2 + 1 = 4x$ **[Ans: $x = \frac{1}{2}, x = \frac{1}{2}$]**
10. $4x^2 - 4x - 35 = 0$ **[Ans: $x = \frac{7}{2}, x = \frac{{}^-5}{2}$]**

Use factorization to solve the following quadratic equations;

11. $x^2 + 2x - 63 = 0$ **[Ans: $x = 7, x = {}^-9$]**
12. $x(x - 2) = 3(x - 2)$ **[Ans: $x = 2, x = 3$]**
13. $x^2 - 5x = 50$ **[Ans: $x = 10, x = {}^-5$]**
14. $3x^2 + x = 2$ **[Ans: $x = \frac{2}{3}, x = {}^-1$]**
15. $2x^2 - 3x - 5 = 0$ **[Ans: $x = \frac{5}{2}, x = {}^-1$]**
16. $(p - 4)^2 = (p - 4)(2p - 1)$ **[Ans: $p = 4, p = {}^-3$]**
17. $q^4 - 10q^2 + 24 = 0$ **[Ans: $q = 2, q = {}^-2, q = \sqrt{6}, q = {}^-\sqrt{6}$]**
18. $(2 + x)^2 + (x - 1)^2 = 5$ **[Ans: $x = 0, x = {}^-1$]**
19. $(x - 1)(x + 2) = 4$ **[Ans: $x = 2, x = {}^-3$]**
20. $2x^2 + 7x - 4 = 0$ **[Ans: $x = \frac{1}{2}, x = {}^-4$]**

Use quadratic formula to solve the following quadratic equations.

21. $(2x - 7)^2 = 1$ **[Ans: $x = 3, x = 4$]**
22. $6x^2 - 23x - 4 = 0$ **[Ans: $x = 4, x = \frac{{}^-1}{6}$]**
23. $3x^2 - 14x - 5 = 0$ **[Ans: $x = 5, x = \frac{{}^-1}{3}$]**
24. $4x^2 - 3x - 10 = 0$ **[Ans: $x = 2, x = \frac{{}^-5}{4}$]**
25. $10x^2 - 3x - 1 = 0$ **[Ans: $x = \frac{1}{2}, x = \frac{1}{5}$]**
26. $6k^2 + 7k + 1 = 0$ **[Ans: $k = {}^-1, k = \frac{{}^-1}{6}$]**
27. The length of a rectangle is 2 *m* more than the width and the rectangle is 63 m^2. Find the dimensions of the rectangle. **[Ans: width = 7 *m*, length = 9 *m*]**
28. The sides of a right angled triangle are $(2x - 3)m$, $(2x + 1)m$ and $(2x + 5)m$. Find the value of *x* and hence find the dimensions of the triangle. **[Ans: *x* = 7.5 *m*, opposite = 12 *m*, adjacent = 16 *m*, hypotenuse = 20 *m*]**
29. The length of a rectagular room exceeds the breadth by 5 *m* and the area of the floor is 50 m^2. Find the breadth of the room. **[Ans: 5 *m*]**
30. The hypotenuse of a right angled triangle is 17 *cm* long and one of the sides containing the right angle is 7 *cm* more than the other.

    (i) Find the length of the shortest side of the triangle. **[Ans: 8 *cm*]**

(ii) Find the area of the triangle. **[Ans: 60 cm^2]**

31. The numbers add up to 20. The sum of their squares is 20 less than 3 times of their product. Find the numbers. **[Ans: 6 and 14]**

32. The co-ordinates of the points P and Q are P(5, 4) and Q($^-3$, k). Find the possible values of k such that $|PQ| = \sqrt{65}$. **[Ans: $k = 3$, $k = 5$]**

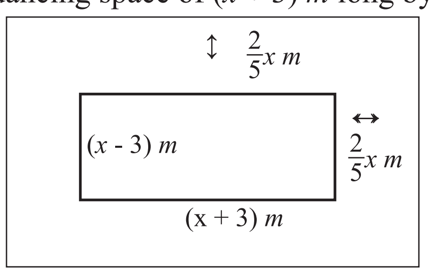
33. If $x = 2$ is a solution of the equation $x^2 + kx - 10 = 0$. Find the value of k and the second value of x. **[Ans: $k = 3$, $x = {}^-5$]**

34. Find the quadratic equations corresponding to the following solution sets.
 (a) $\{^-3, 4\}$ (b) $\{^-4, 6\}$ (c) $\{3.5, {}^-2.5\}$ (d) $\left\{\frac{^-6}{7}, 1\right\}$
 (e) $\left\{\frac{^-4}{3}, \frac{3}{2}\right\}$ (f) $\{^-2, {}^-1\}$ (g) $\left\{\frac{^-3}{2}, 5\right\}$

35. (a) The difference between the squares of two numbers is 40. If the langer number exceeds the smaller by 4, find the two numbers. **[Ans: 7 and 3]**
 (b) A man is now four times as old as his son. Eight years ago, the product of their ages was 220 years. Find the son's present age. **[Ans: 13 years]**

36. Jane is three years older than Peace. In two years time, the product of their ages will be 238. How old is Jane now? **[Ans: 15 years]**

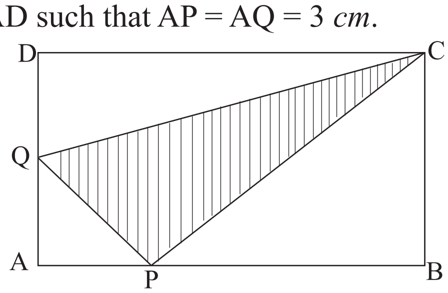
37. The figure below represents the floor of a dining hall with a carpet margin all arround of $\frac{2}{5}x$ wide m wide leaving a dancing space of $(x + 3)$ *m* long by $(x - 3)$ *m* wide.



 (a) If the total area of the entire room is 315 m^2, calculate the value of x. **[Ans: x = 10 *m*]**
 (b) Hence calculate the area of the carpeted margin. **[Ans: 224 m^2]**
 (c) If the carpet costs shs 750 per m^2, calculate the total cost of the carpeted margin. [**Ans**: shs 168,000]

38. The number of diagonals in a polygon = $\frac{n(n-3)}{2}$, where n is the number of sides. How many sides are there if the number of diagonals is;
 (a) 54 **[Ans: 12]** (b) 135 **[Ans: $n = 18$]**

39. In the given figure, ABCD is a rectangle with AB = $2x$ *cm* and BC = x *cm*. P and Q are points on AB and AD such that AP = AQ = 3 *cm*.



 (i) Show that the sum of the curves of triangles PBC and QDC = $\left(2x^2 - \frac{9x}{2}\right)$ cm^2
 (ii) If the area of triangle PCQ = 22.5 cm^2, find x.
 [Ans: $x = 6$ *cm*]

QUADRATIC CURVES

TRIAL QUESTIONS

1. Draw a graph of $y = 2x^2 - 4x - 10$ for values of x from $^-2$ to 4. Use a scale of 1 cm : 1 unit on the vertical axis and 2 cm : 1 unit on the horizontal axis. Use your graph to solve $2x^2 - 4x - 10 = 0$

2. (a) Copy and complete the table below for the function $y = 2 + 3x - 2x^2$ for values of x from $^-4$ to $^+5$.

x	$^-4$	$^-3$	$^-2$	$^-1$	0	1	2	3	4	5
x^2										
$3x$										
$^-2x^2$										
2										
y										

(b) Draw the graph of $y = 2 + 3x - 2x^2$ using a scale of 2 cm : 1 unit on the x – axis and 2 cm : 5 units on the y – axis.

(c) From your graph read the roots of:

(i) $2 + 3x - 2x^2 = 0$ (ii) $7 + 3x - 2x^2 = 0$ (iii) $6 + 9x - 6x^2 = 5x - 15$

3. (a) Draw a graph of $y = (x + 1)(x - 3)$ for the domain $^-2 \leq x \leq 4$. Using a scale of 2 cm : 1 unit on both axes

(b) using your graph find the roots of the equations;

(i) $x^2 - 2x - 3 = 0$ (ii) $x^2 - 3x - 1 = 0$ by drawing

4. (a) Copy and complete the following table if $y = 6 - x - 2x^2$ for $-3 \leq x \leq 3$

x	$^-3$	$^-2.5$	$^-2.0$	$^-1.5$	$^-1.0$	$^-0.5$	0	0.5	1.0	1.5	2.0	2.5	3.0
6	6	6	6	6	6	6	6	6	6	6	6	6	6
^-x													
$^-2x^2$													
y													

(b) Draw the graph of $y = 6 - x - 2x^2$ for $^-3 \leq x \leq 3$ using your values in (a)

(c) Use your graph to find the roots of; (i) $6 - x = 2x^2 = 0$ (ii) $1 + x - 2x^2 = 0$

5. Copy and complete the table below in which $y = x^2 - 6x + 2$

x	$^-1$	0	1	2	3	4	5	6	7
x^2		0			9			36	49
^-6x	6	0	-6			$^-24$			$^-42$
$^+2$		$^+2$		$^+2$					$^+2$
y		2		$^-6$			$^-3$		9

(a) Draw the graph of $y = x^2 - 6x + 2$ for $^-1 \leq x \leq 7$

(b) Use your graph to estimate the roots of the equation $x^2 - 6x + 2 = 0$

(c) Using the same axes, draw the line $y = 4$

(d) Use your graphs to estimate the roots of the equation $x^2 - 6x - 2 = 0$.

6. (a) Using a scale of 1 cm to represent 1unit on the x-axis and 1 cm to present 2 cm on the y-axis, draw the graph of $y = 2x^2 + 3x - 11$ for the range of values of $x = {}^-4$ to $x = 3$.

(b) On the same axes draw the graph of $y = 2x + 1$.

(c) Use your graph to solve the quadratic equations; (i) $2x^2 + 3x - 11 = 0$ (ii) $2x^2 + x - 12 = 0$

7. (a) Draw the graph of $y = x^2 - 3x + 2$ for $^{-}1 \leq x \leq 4$, using a scale of 2 *cm* to represent 1 unit on either axis.
(b) Use your graph in (a) above to solve the following quadratic equations;
(i) $x^2 - 3x + 2 = 0$ (ii) $x^2 - 4x = 0$

8. (a) Draw a graph of $y = 2 + 3x - 2x^2$ for values of x from $^{-}4$ to 6, using a scale of 2 *cm* : 1 unit on the x-axis and 2 *cm* : 5 units on the y-axis.
(b) Use your graph to find the roots of ;
(i) $2 + 3x - 2x^2 = 0$ (ii) $6 + 2x - 2x^2 = 0$
(c) State the maximum value of the function.

9. (a) On the same axes and using scales 1 *cm* to represent 1 unit on x-axis and 2 *cm* to represent 5 units on y-axis draw the graph of $y = 2x^2 + 4x + 9$ for the range of values $^{-}4 \leq x \leq 3$.
(b) State the coordinates of the minimum point
(c) On the same axes draw the line $y = 2x + 3$
(d) Use your graphs to solve the equation $2x^2 + 2x - 12 = 0$

10. (a) Draw the graph of $y = 3x^2 - 3x$ taking values of x in the domain of $^{-}2 \leq x \leq 3$. Using a scale of 1 *cm* to represent 2 units on the vertical axis and 1 *cm* to represent 0.5 units on the horizontal axis.
(b) Use your graph in (a) above to solve the equations
(i) $3x^2 - 3x = 0$ (ii) $3x^2 - x - 10 = 0$

11. (a) Using a suitable table of values, draw the curve $y = 3 + 5x - 2x^2$ for $^{-}3 \leq x \leq 4$.
(b) Use the graph to solve the equations;
(i) $3 + 5x - 2x^2 = 0$ (ii) $2 + 3x - 2x^2 = 0$
(c) State the maximum point and the equation of line of symmetry.

12. (a) Draw the graph of the function $y = x^2 - x - 6$ for $^{-}4 \leq x \leq 5$.
(b) State the least value of the function
(c) Use the graph drawn to solve the solve the quadratic equation;
(i) $x^2 - x - 6 = 0$ (ii) $x^2 - 2x - 8 = 0$

13. Given the equation of a curve y = $2x^2 + 5x - 3$
(i) Copy and complete the table below.

x	$^{-}0.4$	$^{-}3.5$	$^{-}3$	$^{-}2.5$	$^{-}2$	$^{-}1.5$	$^{-}1$	$^{-}0.5$	0	0.5	1
$2x^2$					8					0.5	
$5x$					$^{-}1$					2.5	
$^{-}3$	$^{-}3$	$^{-}3$	$^{-}3$	$^{-}3$	$^{-}3$	$^{-}3$	$^{-}3$	$^{-}3$	$^{-}3$	$^{-}3$	$^{-}3$
y					$^{-}5$					0	

(ii) On the same axes and using the same scales plot the graphs of $y = x + 1$ and $y = 2x^2 + 5x - 3$.
(iii) Using your graph solve the equation $x^2 + 2x - 2 = 0$.

14. Given the curve $y = 2x^2 + 3x$ and the line $y = 5 + 4$, determine the coordinates of the points of intersection of the curve and the line.

15. (a) Complete the table below for the function $y = 2x^2 + 3x - 1$

x	$^{-}4$	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2
$2x^2$					0		
$3x - 1$					$^{-}1$		
y					$^{-}1$		

(b) From the above table draw the graph of the function y = $2x^2 + 3x - 1$ for $-4 \leq x \leq 2$ and use the graph to estimate the roots of the equation;
(i) $2x^2 + 3x - 1 = 0$ (ii) $2x^2 + x - 3 = 0$

16. (a) Plot the graph of $y = 3x^2 + 2x - 16$ for values of x, $^-3 \leq x \leq 3$.
    (b) Use your graph to solve the equation; $3x^2 + 2x - 8 = 0$
17. (a) (i) Draw a table for values of y and x for the curve $y = 10 - x^2$. Use values of x from $^-4$ to $^+4$.
    (ii) Use your table to draw a graph of $y = 10 - x^2$.
    (b) On the same axes, draw the graph of the line $y = 2x + 3$
    (c) Use your graphs to solve the equation $x^2 + 2x - 7 = 0$.
18. (a) Draw the graph of the curve $y + 2x + 3 = x^2$ for values of x from $^-3$ to 5. Using a scale of 1 *cm* to represent 1 unit on both axes.
    (b) Use your graph to find the roots of the equations
    (i) $x^2 - 2x = 3$ (ii) $x^2 - x - 6 = 0$ (iii) $2x^2 - 9x + 4 = 0$
19. (a) Draw the graph of $y = 3x^2 + 4x - 4$ for $^-4 \leq x \leq 4$
    (b) Using your graph solve the equations;
    (i) $3x^2 + 4x - 4 = 0$ (ii) $3x^2 + 4x - 6 = 0$ (iii) $3x^2 + 2x - 5 = 0$
20. (a) Draw a graph of $y = 12 - 2x - 2x^2$ for $^-5 \leq x \leq 4$. Using a scale of 1 *cm* : 2 units for vertical axis and 2 *cm* : 1unit for horizontal axis.
    (b) Using your graph solve the equations;
    (i) $12 - 2x - 2x^2 = 0$ (ii) $10 - 8x - 2x^2 = 0$
    (c) State the equation of a line of symmentry of this curve $y = 12 - 2x - 2x^2$.
21. (a) Draw a graph of $y = 6 - x - x^2$ for values of x from $^-4$ to $^+3$ using a scale of 2 *cm*:1 unit on both axes.
    (b) Use your graph to solve: (i) $6 - x - x^2 = 0$ (ii) $2 - x - x^2 = 0$
    (c) State the range of values of x for which y is positive.
22. (a) Draw the graph of $y = 2x^2 + 3x - 1$ for $^-4 \leq x \leq 2$ and use the graph to estimate the roots of the equation $2x^2 + 3x - 1$ to 1 decimal place. Use a scale of 2 *cm* for 1 unit on the x-axis and 1 *cm* for 2 units on the y-axis.
    (b) By drawing a suitable straight line graph on the plotted function in (a) above, estimate the roots of the equation $2x^2 + x - 3 = 0$.
23. The table below shows values for $y = 5 + 8x - 2x^2$ for $^-1.5 \leq x \leq 6$

x	$^-1.5$	$^-1$	0	1	2	3	4	5	6
$8x + 5$						29			
$^-2x^2$		$^-2$							$^-72$
y				11				$^-5$	

    (a) Copy and complete the table above.
    (b) Draw the graph of $y = 5 + 8x - 2x^2$ for $^-1.5 \leq x \leq 6$
    (c) Use your graph to solve the equation.
    (i) $5 + 8x - 2x^2 = 0$
    (ii) $^-2x^2 + 5x + 3 = 0$
    (d) State the coordinates of the turning point of the curve $y = 5 + 8x - 2x^2$

FUNCTIONS

RECALL

This includes the following;

◊ Domain(object) and Range (image)

◊ Solving functions.

◊ Unddefined functions/meaningless functions / infinite functions. (When denominator of a functions zero,then equate denominator to zero.
When a function is equal to to zero, $f(x) = 0$,(equate the numerator to zero)

◊ Composite functions, This is a combination of two or more functions. e.g $fg(x)$, $hf(x)$

◊ Inverse of a function: $f^{-1}(x)$, $(gf)^{-1}(x)$ etc.

Example 1

Given that $f(x) = \frac{x+1}{2x-4}$, find the value of x for which
(i) $f(x)$ is undefined
(ii) $f(x)$ is equal to zero.

Solution;

(i) for undefined function, the denominator of $f(x)$ is equal to zero.

$f(x) = \frac{x+1}{2x-4}$, then $2x - 4 = 0$

$2x = 4$

$\frac{2x}{2} = \frac{4}{2}$

$x = 2$

(ii) $f(x)$ is equal to zero

$f(x) = \frac{x+1}{2x-4}$, $f(x) = 0$

$0 = \frac{x+1}{2x-4}$

$0(2x - 4) = (x + 1)$

$x + 1 = 0$

$x = {}^-1$

Example 2

If $g(x) = \frac{4x+9}{x+4}$, find the inverse of $g(x)$ $[g^{-1}(x)]$

Solution

Let any letter be equal to the given function.

$y = \frac{4x+9}{x+4}$, then make x the subject of the formula

$\frac{y}{1} = \frac{4x+9}{x+4}$

$y(x + 4) = 1(4x + 9)$

$yx + 4y = 4x + 9$

collect like terms

$yx - 4x = 9 - 4y$

$x(y - 4) = 9 - 4y$

$x = \frac{9-4y}{y-4}$

Then substitute the value selected back to x

$f^{-1}(x) = \frac{9-4x}{x-4}$

Example 3

If $f(x) = 2x + 3$ and $g(x) = 6x$, find (i) $fg(x)$ (ii) $gf(x)$

Solution

(i) $f(x) = 2x + 3$, $g(x) = 6x$

$fg(x)$ means that where there is x In $f(x)$, substitute $g(x)$

$fg(x) = 2(g(x)) + 3$

but $g(x) = 6x$

$fg(x) = 2(6x) + 3$

$fg(x) = 12x + 3$

(ii) $gf(x)$: This means that, where there is x In $g(x)$, substitute $f(x)$

$gf(x) = 6(f(x))$

$= 6(2x + 3)$

$gf(x) = 12x + 18$

TRIAL QUESTIONS

(1) Given that $g(x) = x^3 + 1$, find the values of

(a) $g(0)$, (b) $g(5)$ (c) $g\left(\frac{3}{4}\right)$, (d) $g({}^-2)$

[Ans:(a) 1, (b) 126, (c) $\frac{91}{64}$, (d) ${}^-7$]

(2) The domain of the function $f(x) = 5x + 1$, is $\{0, 1, 2, 3, 4, 5\}$. Find its range. **[Ans, {1, 6, 11, 16, 21,26}]**

(3) If $f(x) = 7x$ and $g(x) = x + 3$ and $fg : x_1 \rightarrow y$, express as simply as possible the rule which maps x onto y.
[Ans: $7x + 21$]

Find the values of p, q, r such that; (a) $fg{:}5 \rightarrow p$ (b) $fg{:}10 \rightarrow q$, (c) $fg{:}r \rightarrow 35$

[Ans: $p = 56$, $q = 91$, $r = 2$]

Find also the function (F) which reverses the function fg, that is it maps y onto (back to) x. **[Ans: $\frac{x-21}{7}$]**

(4) If $f(x) = ax + b$ and $f(2) = 10$, while $f(^-3) = 5$. Find

(i) value of a and b **[Ans: $a = 1, b = 8$]**

(ii) $f^{-1}(x)$ **[Ans: $x - 8$]**

(5) (a) If $h(x) = nx + 7$ and $h(5) = 22$, find;

(i) the value of n **[Ans: $n = 3$]**

(ii) $h(4)$ **[Ans: 19]**

(b) Given that $f(x) = 8x^2 + 5$, find

(i) $f(^-3)$ **[Ans: 77]**

(ii) $f(2)$ **[Ans: 37]**

(iii) The value of x for which $f(x) = 293$ **[Ans: $x = \pm 6$]**

(6) If $f^{-1}(x) = 2x^2 - 3$, find

(i) an expression for $f(x)$ **[Ans: $f(x) = \sqrt{\frac{x+3}{2}}$]**

(ii) $f(15)$ **[Ans: 3]**

(7) If $h(x) = 4 - x^2$ and $g(x) = 2x + 1$,

Find (i) $gh(x)$ **[Ans: $9 - 2x^2$]**

(ii) $gh(^-2)$ **[Ans: 1]**

(8) (a) Given that $g(x) = px + q$, $g(2) = 6$ and $g(^-4) = 3$. Find the values of p and q. **[Ans: $p = \frac{1}{2}$, $q = 6$]**

(b) If $f(x) = \frac{1}{x}$, find $\frac{1}{f^2(x)}$ for $x = 1, 2, 3,$ and 4 **[Ans: 1, 4, 9, 16]**

(9) (a) Given that $f(x) = 3x - 2$, find $f^{-1}(x)$ and $f^{-1}(10)$. **[Ans: $\frac{x+2}{3}$, 4]**

(b) If $h(x) = rx^2 + t$, $h(1) = 7$ and h(0) = 2, find the values of r and t. **[Ans: $t = 2, r = 5$]**

(10) (a) Given that $f(x) = \frac{1-x^2}{4x^2-1}$, find the values of x for which

(i) $f(x) = 0$ **[Ans: $x = \pm 1$]**

(ii) $f(x)$ is undefined **[Ans: $\pm \frac{1}{2}$]**

(b) If $f(x) = \frac{3x}{2x+1}$, find $f^{-1}(x)$ Hence calculate $f^{-1}(2)$ **[Ans: $\frac{x}{3-2x}$, $^-2$]**

(11) (a) Given that $f(x) = px - q$, $f(2) = 10$ and $f(1) = 7$, find the values of p and q. **[Ans: $p = 3, q = {^-4}$]**

(b) If $g(x) = \frac{2x-1}{x^2-9}$, find the values of x for which the function $g(x)$ is undefined. **[Ans: $x = \pm 3$]**

(c) Given that $f(x) = x^2 + 7$ and $g(x) = x - 2$, find the value of x for which $fg(x) = \frac{38 - x^2}{2}$. **[Ans: $x = 4$ or $x = \frac{^-4}{3}$]**

(12) (a) Given that $f(x) = 3x + 5$ and $g(x) = \frac{2}{x-5}$, find

(i) $gf(x)$ **[Ans: $\frac{2}{3x}$]**

(ii) $gf\left(\frac{1}{2}\right)$ **[Ans: $\frac{4}{3}$]**

(b) If $h(x) = px + 3$ and that $h(4) = 23$, find the value of

(i) p (ii) $h(0)$ (iii) $h(^-5)$ **[Ans: (i) $p = 5$, (ii) 3, (iii) $^-22$]**

(c) Determine;

(i) $h^{-1}(x)$

(ii) $h^{-1}(13)$ **[Ans: (i) $\frac{x-3}{5}$, (ii) 2]**

(13) Given that the function $f(x) = \frac{x}{x-2}$ and $g(x) = \frac{1}{2x-3}$

(a) Find the value of x for which $f(x) - g(x) = \frac{8}{15}$, **[Ans: $x = {^-1}$, $x = \frac{9}{7}$]**

(b) The functions f and g are defined by $f(x) = \frac{x}{x-5}$ and $g(x) = x + 4$. Find;

(i) $g(^-10)$ **[Ans: $\frac{2}{3}$]** (ii) $f^{-1}(x)$ and hence $f^{-1}(6)$ **[Ans: $\frac{5x}{x-1}$, 6]**

(iii) the value of x for which $gf(x) + fg(x) = 0$ **[Ans: $x = \frac{13}{3}$ or $x = 0$]**

(14) (a) Given that $f(x) = 2x + 6$ and $g(x) = \frac{x}{3x-10}$. Find;

(i) $gf(x)$ (ii) $gf(\frac{1}{3})$ **[Ans: (i) $\frac{x+3}{3x+4}$ (ii) $\frac{2}{3}$]**

(b) A function h is defined by $h(x) = \frac{x+4}{3x+2}$, find

(i) $h^{-1}(x)$

(ii) The value of x for which $h^{-1}(x)$ is meaningless. **[Ans: (i) $\frac{4-2x}{3x-1}$, (ii) $x = \frac{1}{3}$]**

(15) (a) Given that $g(x) = (x + 2)(x - 3)$ find

(i) $g(5)$ **[Ans: 14]**

(ii) the values of x for which $g(x) = 0$ **[Ans: $x = {}^-2, x = 3$]**

(iii) the range of values of x for which $f(x) < 0$ **[Ans: $f{:}x \rightarrow x \leq {}^-4$]**

(b) Given that $f(x) = 2x + 9$ and $g(x) = 2 - x$, write an expression for $gf(x)$. Hence find the value of $gf(3)$. **[Ans: $^-7 - 2x$, $^-13$]**

(16) (a) Given that $g(x) = \frac{3x}{x+4}$ and $h(x) = x^2 - 2$. Find

(i) $g(^-5)$ **[Ans: 15]**

(ii) $gh(2)$ **[Ans: 1]**

(b) Find the value of a, given that $f(x) = \frac{2x+a}{4-x}$ and $f(0) = 1$. **[Ans: $a = 1$]**

(17) (a) A function $f(x)\ x - m$ and $f^{-1}(x) = 2xk - 7$ where m and k are constants. Calculate the values of m and k. **[Ans: $k = \frac{1}{2}$, $m = {}^-7$]**

(b) Given that $h(x) = px^2 - qx + 1$, $h(2) = 11$ and $g(1) = 2$. Find the values of p and q. **[Ans: $p = 4$, $q = 3$]**

(c) Given that $f(x) = \frac{x+5}{6}$ and $fg(x) = \frac{7-x}{2}$. Find;

(i) $f(^-17)$ **[Ans: $^-2$]**

(ii) an expression for $g(x)$ and hence evaluate $g(4)$ **[Ans: $g(x) = 16 - 3x$, $g(4) = 4$]**

(18) (a) Determine the range corresponding to the domain $\{^-3, ^-2, 0, 1,, 3, 4\}$ for the mapping $x \rightarrow x^2 + 1$. **[Ans: {10, 5, 1, 2, 10, 17}]**

(ii) Represent the mapping in (i) above on an arrow diagram.

(iii) State the mapping in (ii) above. **[Ans: One to many]**

(b) Given that the functions $h(x) = x + 2$, $g(x) = x^2$ and $f(x) = {}^-x$, find the values of x for which $gh(x) = f(x)$. **[Ans: $x = {}^-1$ or $x = {}^-4$]**

(19) (a) The function h is defined as $h(y) = my^2 - 2$. Given that $h(3) = 16$, find;

(i) the value of m **[Ans: $m = 2$]**

(ii) $h^{-1}(6)$ **[Ans: ±2]**

(b) Given that $f(x) = x + 4$, $g(x) = 3x^2$, find the value of x for which $gf(x) = fg(x)$. **[Ans: $^-\frac{11}{6}$]**

(20) (a) Given that $f(x) = \frac{1}{2x-6}$, find

(i) $f^{-1}(x)$ (ii) $f^{-1}(^-1)$ **[Ans: (i) $\frac{1+6x}{2x}$, (ii) $\frac{5}{2}$]**

(b) Given that $f(x) = x - 5$, $g(x) = x^2$ and $h(x) = \frac{x+2}{6x^2+x-2}$, find $f(^-6)$. **[Ans: $^-11$]**

(c) Find the value of x for which

(i) $gf(x) = fg(x)$ **[Ans: $x = 3$]**

(ii) $h(x)$ is not defined **[Ans: $x = \frac{1}{2}$ or $x = \frac{^-2}{3}$]**

(21) (a) Given that function $f(x) = \frac{8-x}{5}$ and $g(x) = \frac{9+24x+8x^2}{10}$, find the values of x for which $f(x) - g(x) = 0$. **[Ans: $x = \frac{^-7}{2}$, $x = \frac{1}{4}$]**

(b) If the functions $f(x) = \frac{x+3}{2}$ and $g(x) = \frac{1-2x}{5}$, determine the value of x for which $fg(x) + gf(x) = 0$. **[Ans: $x = 3$]**

(22) (a) Draw an arrow diagram for the function $f(x) = 2x^2 + 2$ given that $^-2 \leq x \leq 2$.

(b) Given that $f(x) = \frac{(x+2)(x-3)}{(x-5)}$, find;

(i) $f(^-5)$ **[Ans: $^-2.4$]**

(ii) the values of x for which $f(x) = 0$. **[Ans: $x = {}^-2, x = 3$]**

(iii) The values of x for which $f(x)$ is meaningless. **[Ans: $x = 5$]**

(c) Given that $f(x) = x^2 + 1$ and $h(x) = x - 3$, find the value of x for which $fh(x) = hf(x)$. **[Ans: $x = 2$]**

(d) The function $h(x) = \frac{4}{5x^2 - 12x + 4}$, find the value of x for which $h(x)$ is meaningless. **[Ans: $x = \frac{2}{5}, x = 2$]**

(23) (a) Given that $f(x) = ax + b$, $f(4) = 4$ and $f(^-1) = {}^-6$. Find the values of a and b. **[Ans: $a = 2, b = {}^-4$]**

(b) Given that $g(x) = \frac{2}{x+2} + \frac{3x+4}{x^2-4}$, express $g(x)$ in the form $\frac{px}{x^2+q}$, where p and q are integers. State the values of x for which $g(x)$ is not defined. [$p = 5, q = {}^-4$] **[Ans: $x = \pm 2$]**

(c) The function $h(x) = \frac{4}{5x+1}$ find;

(i) $h^{-1}(x)$ (ii) $h^{-1}(2)$. **[Ans: (i) $\frac{4-x}{5x}$ (ii) $\frac{1}{5}$]**

(24) Given that $f(x) = \frac{x+5}{2}$ and $g(x) = \frac{1-3x}{3}$, determine the values of x for which $fg(x) = \frac{x^2+2x-20}{6}$.
[Ans: $x = 4$ and $x = {}^-9$]

(25) (a) Given f is a function defined by $f: x \rightarrow 2x^2 - 1$, find $f(x+3)$ and hence $f(^-2)$.
[Ans: $f(x+3) = 2x^2 + 12x + 17, f(^-2) = 7$]

(b) Given a function $f(x) = \frac{3-x}{x^2-4}$. Find the value of x for which

(i) $f(x) = 0$ **[Ans: $x = 3$]**

(ii) $f(x)$ is undefined. **[Ans: $x = 2, x = {}^-2$ or $x = \pm 2$]**

(26) Given $g(x) = 2x - 1$,write an expression for gg(x) hence determine the value of x for which gg(x) = 17.
[Ans: $gg(x) = 4x - 3, x = 5$]

(27) Given $f(x) = 2 - x$ and $g(x) = \frac{3}{x+1}$

(a) Write expressions for

(i) $gf(x)$ **[Ans: $\frac{3}{3-x}$]** (ii) $fg(x)$ **[Ans: $\frac{2x-1}{x+1}$]**

(b) Determine the value of x for which

(i) $fg(x) + gf(x) = 0$ **[Ans: $x = 0$ or $x = 5$]**

(ii) $gf(x)$ is meaningless **[Ans: $x = 3$]**

(28) (a) Given that $f(x) = \frac{x^2-1}{5}$, find;

(i) $f^{-1}(x)$ **[Ans: $\sqrt{5x+1}$]**

(ii) $f^{-1}(16)$ **[Ans: 9]**

(b) If $g(x) = x^2 - 2x - 1$ and $h(x) = x + 3$, find the value of x for which $hg(x) - gh(x) = 2$ **[Ans: $\frac{^-1}{3}$]**

VARIATION AND PROPORTIONALITY

RECALL

Variation means a change in form, position, state, quantity or quality of a thing.
Variation is categorise as follows;

◊ Direct variation/ direct proportionality.
◊ Inverse variation/ inverse proportionality.
◊ Joint variation
◊ Partial variation.

Under direct and inverse variation/ proportionality, one variable depends on one other.

Direct proportionality.

Circumference of a circle is proportional to its diameter. This is usually expressed in the form of an equation.

$C = \pi d$

We say C varries as its diameter d.

$C \alpha d$

α - proportional sign.

This means that increse in diameter increases the circumference. To form an equation, we substitute the proportionality sign with an equal sign and introduce the constant of proportionality.

$C = kd$
$k \rightarrow$ constant of proportionality
But with this case $k = \pi$

$C = \pi d$

NB: The word "varies as" is the same as direct proportionality.

Example Given that y varies directly as the square of x. when $y = 8$, $x = 4$. Find the value of y when $x = 2$. **Solution;** $y \alpha x^2$ $y = kx^2$ $8 = k \times 4^2$ $8 = 16k$	$k = \frac{8}{16}$ $k = \frac{1}{2}$ $y = \frac{1}{2}x^2$ when $x = 2$, $y = ?$ $y = \frac{1}{2}(2)^2$ $y = \frac{1}{2} \times 4$ $y = 2$

INVERSE PROPORTIONALITY

One variable increases with decrease in the other and vice versa.

Example Given that p varies inversely as the square of q and that $p = 9$ when $q = 4$, find the value of p when $q = 8$ **Solution;** $p \alpha \frac{1}{q^2}$ $p = \frac{k}{q^2}$	when $p = 9$, $q = 4$ $9 = \frac{k}{4^2}$ $k = 9 \times 4^2$ $k = 144$ $p = \frac{144}{q^2}$	when $q = 8$, $p = ?$ $p = \frac{144}{8^2}$ $p = \frac{144}{64}$ $p = 2.25$

JOINT VARIATION

One variable depends on two or more others eg volume (V) of a right circular cylinder is given in terms of its radius r and height (h) by formular $V = \pi r^2 h$.

$$V \alpha\, r^2 \qquad V \alpha\, h$$
$$V \alpha r^2 h$$
$$V = kr^2 h \text{ but } k = \pi$$
$$V = \pi r^2 h$$

But under this section, both inverse and directly proportionality can be combined together.

Example Given that y varies directly as the square of x and inversely as s. When $y = 8$, $x = 4$ and s = 6. Find y when $x = 6$ and $s = 13.5$. **Solution;** $y \alpha x^2$ $y \alpha \frac{1}{s}$ $y \alpha \frac{x^2}{s}$	$y = \frac{kx^2}{s}$ NB: The first values ie $y = 8$, $x = 4$, $s = 6$ are lood to get the constant $y = \frac{kx^2}{s}$ $8 = \frac{k \times 4^2}{6}$ $8 \times 6 = 16k$	$k = \frac{8 \times 6}{16}$ $k = 3$ $y = \frac{3x^2}{5}$ $y = ?, x = 6, s = 13.5$ $y = \frac{3 \times 6^2}{13.5}$ $y = 8.$

PARTIAL VARIATION

This is the type of variation that involves two constants and these are found by forming two simultaneous equations and solving them using a suitable method. ie elimination, substitution and matrices to find the value of constants.

N.B: If a value has its own constant.

Example P varies partly as the square of V and partly as the cube of V. When V = 2, P = -20 and when V = -3, P = 135. Find the relationship between P and V. Find the value of P when V = -1. **Solution;** $P \alpha V^2 + V^3$ $P = k_1V^2 + k_2V^3$ when $V = 2, P = -20$ $-20 = k(2)^2 + k_2(2)^3$ $4k + 8k_2 = -20$ reduce by 4 $k_1 + 2k_2 = -5$ .... (i) when $V = -3, P = 135$ $P = k_1V^2 + k_2V^3$	$135 = k_1(-3)^2 + k^2(-3)^3$ $135 = 9k_1 + k_2(-27)$ $9k_1 - 27k_2 = 135$ reduce by 9 $k_1 - 3k_2 = 15$ ....(ii) Solve (i) and (ii) simultaneously $k_1 + 2k_2 = -5$ ....(i) $k_1 - 3k_2 = 15$ ....(ii) (i) - (ii) $k_1 + 2k_2 = -5$ $k_1 - 3k_2 = 15$ $0 + 2k_2 - (-3k_2) = -5 - 15$ $5k_2 = -20$ $k_2 = \frac{-20}{5}$ $k_2 = -4$	Substitute k_2 into $k_1 - 3k_2 = 15$ $k_1 - 3(-4) = 15$ $k_1 + 12 = 15$ $k_1 = 15 - 12$ $k_1 = 3$ $k_1 = 3, k_2 = -4$ $P = 3V^2 - 4V^3$ $P = ?, V = -1$ $P = 3(-1)^2 - 4(-1)^3$ $P = 3 - 4(-1)$ $P = 3 + 4$ $P = 7$

TRIAL QUESTIONS

(1) The volume v of a cylinder varies directly as its height h. If the volume V = 50 cm^3 when H = 5 cm. Find thee volume of the cyclinder when H = 20 cm, **[Ans: V = 200 cm^3]**

(2) Given that y is directly proportional to x^2. If $y = 16$ when $x = 1$, find x when $y = 1$. **[Ans: $x = \frac{1}{4}$]**

(3) The force (F) which acts between two magnetic poles is inversely proportional to the square of the distance (d) between them. If F = 18 when $d = 4$, find F when $d = 3$. **[Ans: 32]**

(4) Given than V is inversely proportional to t and V = 25 when $t = 2$. Find the value of V when $t = 5$.
[Ans: V = 10]

(5) Given that y is directly proportional to x^3 and that y is 250 when $x = 10$. Find the equation connecting x and y, hence find the value of y when $x = 4$. **[Ans: $y = 16$]**

(6) (a) If y varies directly as the cube root of x, and that when $x = {}^-8$, $y = {}^-4$. Find x when $y = {}^-8$.

(b) Given that w is inversely proportional to the square of h and that $w = h = {}^-2$. Find h when $w = \frac{-1}{2}$
[Ans: $h = 4$]

(c) The volume V of water oozing out of a narrow water pipe varies as the square of its diameter D and inversely as the reciprocal of the length L of the pipe.

(i) Write down a formula connecting V, D, L and the constant of proportionality k.

(ii) When D = 10 cm, L = 0.3m and V = 300 cm^3. Find k and determine the value of D when V = 1000 cm^3 and L = 40 m. **[Ans: $k = 0.1$, D = 50 cm]**

(7) A quantity T is partly constant and partly varies as the square of q. When $q = 2$, T = 40. when $q = 3$, T = 65.

(a) Form an equation relating T and q. **[Ans: $T = 20 + 5q^2$]**

(b) Determine the values of q when T = 100. **[Ans: $q = 4$]**

(8) A quantity M is partly constant and partly varies as the square of N. When N = 4, M = 95. When N = 3, M = 60.

(a) Form an equation relating M and N. **[Ans: $M = 15 + 5N^2$]**

(b) Determine the values of N when M = 140. **[Ans:N = 5]**

(9) The expenditures (E) of Luwero high school is directly proportional to income (Y) collected in form of school fees and inversely proportional to square number of children (n). When E = 540,000/=, Y = 1,800,000/= and $n = 30$. Find E when Y = 3,000,000/= and $n = 50$. **[Ans: 324,000/=]**

(10) x varies directly as the square of y and inversely as z. Given that $x = 6$, $y = 4$, and $z = 2$. Find z when $x = 8$ and $y = 6$. **[Ans: $\frac{27}{8}$]**

(11) x is inversely proportional to y. $x = 4$ when $y = 5$. Find

(a) x when $y = 2.5$. **[Ans: $x = 8$]**

(b) y when $x = 1.25$. **[Ans: $y = 16$]**

(12) A quantity B varies as the square of V and is inversely proportional to R. When B = 27, V = 6 and R = 4. Determine the value of B when V = 8 and R = 16. **[Ans: B = 12]**

(13) The length (L) of a simple pendulum varies as the square of the period (T). A pendulum with length 0.994 m long has a period of approximately 2 s. Find

(a) an equation connecting L and T. **[Ans: $L = 0.2485T^2$]**

(b) the length of a pendulum whose period is 3 s. **[Ans: L = 2.237 m]**

(14) The distance of the horizon D km varies as the square root of the height H m of the observer above sea level. An observer at a height of 100 m above sea level sees the horizon at a distance 35.7 km, Find;

(a) an equation connecting D and H. **[Ans: $D = 3.57\sqrt{H}$]**

(b) Find the distance of the horizon from an observer 70 m above sea level. **[Ans: D = 29.87 km]**

(15) The cost (C) of printing a circular on octavo paper is partly constant and partly varies as the number of copies (n) printed. If 100 and 500 copies cost £8.25 and £14.25 respectively.

(a) Find an equation connecting C and n. **[Ans: $C = 6.75 + 0.015n$]**

(b) Find how much will 200 copies copies? **[Ans: £9.75]**

(c) Find how many copies produced at a cost of £12.99. **[Ans: 416 copies]**

(16) (a) Given that x varies directly as y and inversely as the square of z and that $x = y$ when $z = 3$. Calculate the value of x when $y = 5$ and $z = 2$. **[Ans: $x = 11.25$]**

(b) The formula $d = kV + mV^2$ gives the distance d metres travelled by a certain car is being brought to rest from a speed Vms-1 by the application of breaks. If $d = 62$ *m* when $V = 40$ ms^{-1} and $d = 117$ *m* when $V = 60$ ms^{-1}.

(i) Find the values of the constants k and m. **[Ans: $k = 0.75$, $m = 0.02$]**

(ii) If the breaks are applied when the car's speed is 50 ms^{-1}, Find the distance it travels before coming to rest. **[Ans: $d = 87.5$ *m*]**

(17) The annual cost (C) of running a certain car is partly constant and partly varies as the distance (D) run by the car in the year. In one year the car ran 6000 *km* at a total cost of £900. In the next year, it ran 7200 *km* at a total cost of £950.

(a) Find an equation connecting cost (C) and distance (D). **[Ans: C = 648 + 0.042D]**

(b) How much would it cost to run the car in a year during which it ran 12000 *km*? **[Ans: £1152]**

(18) Given that x varies directly as y and inversely as the square of Z and that $x = 2$, $y = 2$ and $Z = 3$. Calculate the value of x when $y = 5$ and $Z = 2$.

(b) Given that d partly varies as V and partly varies as the square of V and that $d = 62$ when $V = 40$ and $d = 117$ when $V = 60$, find the value of d when $V = 50$.

(19) The cost (C) of printing a copy of a newspaper is partly constant and is also inversely proportional to the number (n) of copies printed. When 200 copies are printed the cost per copy is shs 850. When 300 copies are printed the cost per copy is shs 750.

(a) Form an equation relating C and n.

(b) Calculate the cost per copy when 150 copies are printed.

(20) Time t hours taken to dig a pit latrine partly varies as the depth (d- metres) of the latrine and partly as the square of the depth. If $t = 80$, $d = 20$ and when $t = 150$, $d = 30$.

(a) Write down an expression connecting t and d. **[Ans: $t = 2d + 0.1d^2$]**

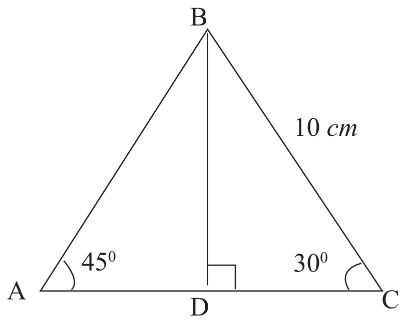
(b) Find t when $d = 40$. **[Ans: $t = 240$]**

TRIGONOMETRY

TRIAL QUESTIONS

1. (a) The height of a right cone is 12 *cm* and the angle at the vertex of the cone is 30^0. Find the volume of the cone correct to 2 decimal places. (Take $\pi = \frac{22}{7}$) **[Ans: 41.47 cm^3]**

   (b)

   The figure shows triangle ABC in which $\overline{BC} = 10$ *cm*, $\angle BCA = 30^0$ and $\angle BAC = 45^0$. Line $\overline{BD}$ is perpendicular to $\overline{AC}$. Find;

   (i) $\overline{BD}$ [**Ans**: 5 *cm*] (ii) $\overline{DC}$ **[Ans: 8.66 *cm*]**

   (iii) Area of the triangle ABC **[Ans: 34.15 cm^2]**

2. If $\sin\theta = \frac{3}{5}$ and $0^0 \leq \theta \leq 360^0$, find the values of $\cos\theta + \tan\theta$ without using tables or calculator.
   [Ans: $\frac{31}{20}, \frac{^-31}{20}$]

3. Given that $\tan\theta = \frac{^-12}{5}$ and $0^0 \leq \theta \leq 180^0$, find without using tables or calculators the value of $\sin\theta + \cos\theta$. **[Ans: $\frac{^-7}{13}$]**

4. The angle of depression of a boat from the top of a vertical cliff. 50 *m* high, is 10^0. Find the distance of the boat from the foot of the cliff. **[Ans: 283.56 *m*]**

5. (a) Given that $\tan\theta = \frac{5}{12}$, find without using tables or a calculator the value of $4\cos\theta - 3\sin\theta$ **[Ans: $\frac{33}{13}$]**

   (b) Given that $\tan\theta = \frac{3}{4}$ and that $0^0 \leq \theta \leq 360^0$, find the values of $\sin\theta + \cos\theta$. **[Ans: $\frac{7}{5}, \frac{^-7}{5}$]**

6. A man of height 1.7 *m* tall is standing 12 *m* away from the foot of a tree. When he looks up at the top of the tree, the angle of elevation is 48^0. Determine the height of the tree, correct to two decimal places. **[Ans: 15.03 *m*]**

7. (a) Building A is 40 *m* high. The angle of depression of the top of building B from the top of A is 26^0. If the two buildings are 10 *m* apart, find the height of building B. **[Ans: 35.12 *m*]**

   (b) Given that $\tan\theta = \frac{^-12}{5}$ and $0^0 < \theta < 180^0$, find without using tables or calculator the value of $3\sin\theta + 2\cos\theta$. **[Ans: $\frac{^-9}{13}$]**

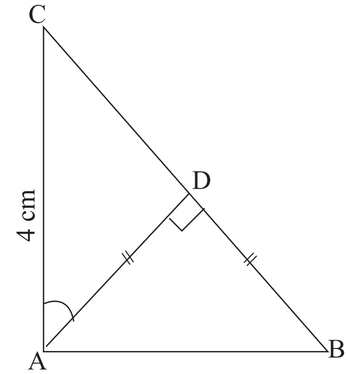
   (c) Given that $\sin\alpha = \frac{8}{17}$ and that $0^0 < \alpha < 90^0$, find without using tables or calculator the value of; $2\cos(90^0 - \alpha) - \sin\alpha$. **[Ans: $\frac{8}{17}$]**

8. (a) If $\sin\theta = \frac{4}{5}$ and $\sin\alpha = \frac{12}{13}$ where $0^0 < \theta < 90^0$ and $90^0 < \alpha < 180^0$, find the value of $\sin\theta\cos\alpha + \cos\theta\sin\alpha$. **[Ans: $\frac{16}{65}$]**

   (b) Given that $\tan\theta = -\frac{3}{4}$ and $0^0 \leq \theta \leq 360^0$, find the possible values of $\cos\theta - \sin\theta$. **[Ans: $\frac{^-7}{5}, \frac{7}{5}$]**

   (c) A students stands 16 *m* away from a vertical flag mast 10.5 *m* high. Find the angle of elevation of the top of the mast from the foot of the student. [**Ans**: 33.3^0]

9. In the figure below AD is perpendicular to BC, AD = DB. AC = 4 *cm* and angle CAD = 30^0. Find the length AB.



[Ans: 4.9 *cm*]

10. (a) Using an interval of 15^0 draw the table of values for $0^0 \leq x \leq 120^0$ of the function (giving your values correct to 2d.p)

    (i) $y = 3\cos x$ (ii) $y = 4\sin(2x - 10^0)$.

    (b) Taking 1 *cm* to represent 15^0 on x-axis and 2 *cm* to represent 1 unit on y-axis, draw the graphs of $y = 3\cos x$ and $y = 4\sin(2x - 10^0)$ on the same axes.

    (c) Using your graphs, find the values of x for which $3\cos x = 4\sin(2x - 10^0)$

11. A man is standing 12 *m* away from the foot of a tree of height 16 *m*. When he looks up at the top of the tree, the angle of elevation is 50^0. Determine the height of the man, correct your answer to two decimal places. **[Ans: 1.7 *m*]**

12. Given that $\tan\theta = \frac{-3}{4}$ and $0^0 \leq \theta \leq 270^0$ without using tables or calculator evaluate $10\sin\theta - \frac{1}{4}\cos\theta$. **[Ans: $\frac{29}{5}$, $\frac{-29}{5}$]**

13. Given that $\cos\theta = \frac{1}{\sqrt{3}}$ and θ lies in the first quadrant, find the value of $\frac{\tan\theta + \sin\theta}{\cos\theta}$ in its simplest form. (Leave your answer in surds). **[Ans: $\sqrt{6} + \sqrt{2}$]**

14. (a) Copy and complete the table below.

x^0	0^0	5^0	10^0	15^0	20^0	25^0	30^0	35^0	40^0	45^0	50^0	55^0	60^0
$\frac{3}{2}\tan x$	0												
$2\cos x - 1$	1.00			0.93									

    (b) Draw the graphs for the functions $y = \frac{3}{2}\tan x$ and $y = 2\cos x - 1$ on the same axes. Use a scale of 1 *cm* to represent 5^0 on the x-axis and 2 *cm* to represent 1 unit on the y-axis.

    (c) Use your graph to solve the equation $3\tan x + 2\cos x$

15. (a) Given that $\tan x = \frac{-3}{4}$ and $0° \leq x \leq 360^0$, Without using tables or calculator, find the possible values of $\cos x + \sin x$. **[Ans: $\frac{-1}{5}$, $\frac{1}{5}$]**

    (b). Given that $\sin x = 0.6$ and x is obtuse. Without using tables or calculator, find;

    (i) $\cos x$ **[Ans: $\frac{-4}{5}$]** (ii) $\tan x$ **[Ans: $\frac{-3}{4}$]**

    (c). Given that $\cos x = \frac{3}{5}$ and x is a reflex angle,without using tables or calculators,find;

    (i) $\sin x$ (ii) $\tan x$ **[Ans: (i) $\frac{-4}{5}$, (ii) $\frac{-4}{3}$]**

16. From the top of the tower, the angle of depression of a flower bed is 52^0. If the tower is 42 *m* high, find the distance from the tower to the flower bed. **[Ans: 32.81 *m*]**

17. A girl is looking at the top of a tree. The girl is 2 *m* tall and the angle of elevation of the bird on top of a tree from the girl is 30^0. How high is the tree if the girl is 30 *m* away from the tree? **[Ans: 19.32 *m*]**

18. A boy is looking at the top of the building which is on the ground that levels with him. He measures the angle of elevation of the top of the building 32^0. He walks 30 *m* towards the building and finds that the angle of elevation of the building is now 45^0. The boy is 2.3 *m* tall. How far was he from the building when he started? **[Ans: 80 *m*]**

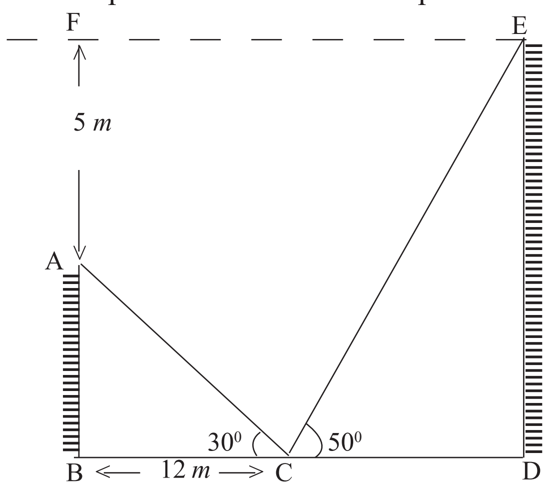
19. At a certain point on the ground, the angle of elevation of the top of the tower T is 28^0. At another point 100 *m* away from the first point, the angle of elevation is 35^0. Find the two expressions for the height of the tower and hence find the height. **[Ans: 221 *m*]**

20. The angles of depression of the two boats from the top the cliff are 39^0 and 60^0. The boats are in a straight line from the point of observation. How far apart are the two boats if the cliff is 400 *m* high? **[Ans: 263.02*m*]**

21. (a) Draw the graph of the curve $y = \cos 3x$ for $0 \leq x \leq 150^0$.

    (b) Using your graph,determine the vales of x fo r which $4\cos 3x + 3 = 0$.

22. In the diagram, two buildings AB and DE are shown. DE is 5m higher than AB. The angles of elevation of the tops A and E from a fixed point C are 30^0 and 50^0 respectively. BC is 12 *m* from C,



Find;

(i) Height of AB **[Ans: 6.93 *m*]**

(ii) Height of DE. **[Ans: 11.93 *m*]**

(iii) The distance BD between the two buildings **[Ans: 22 *m*]**

23. The angle of elevation of the top of a cliff from a point A due East of it and 90 *m* away from its base us 30^0. From another point B due West of the cliff, the angle of elevation of the top of the cliff is 60^0.

    (a) calculate height of the cliff. **[Ans: 51.96 *m*]**

    (b) Calculate how far B is from the base of the cliff. **[Ans: 30.0 *m*]**

    (c) How far is point B from the top of the cliff. **[Ans: 60.0 *m*]**

24. Two men 4 *m* and 3 *m* tall stand on two opposite sides of a mast, which is 30m tall. The angle of elevation of the top of the mast from each of the men is 45^0. Calculate:

    (i) how far apart the two men are on the ground. **[Ans: 60 *m*]**

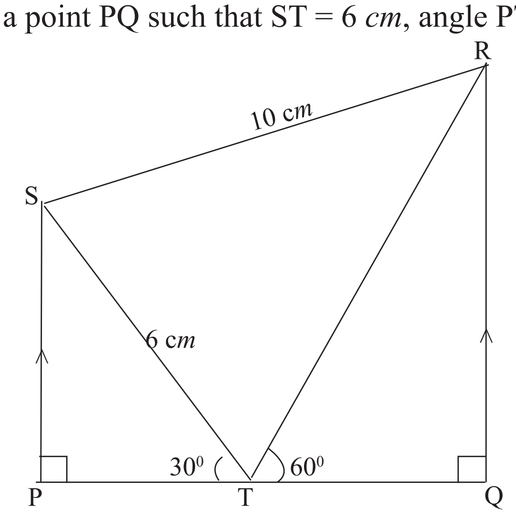
    (ii) the angle of depression of the shorter man from the taller man. **[Ans: 3.8^0]**

25. (a) At a certain point on the level ground the angle of elevation of the top of a tower, T, is 30^0. At another point 100 *m* away from the first point, the angle of elevation is 35^0.

    (i) Find two expressions for the height of the tower

    (ii) Hence find the height of the tower to the nearest metre.

26. The figure below shows a trapezium in which PS is parallel to QR, SR = 10 *cm* and angle PQR = 90^0. T is a point PQ such that ST = 6 *cm*, angle PTS = 30^0 and angle QTR = 60^0.



(a) Calculate; leaving your answers in surd form where applicable the lengths:

(i) TR. **[Ans: 8]** (ii) QR. **[Ans: $4\sqrt{3}$]**

(iii) QT **[Ans: 4]** (iv) PS **[Ans: 3]**

(v) PT **[Ans: $3\sqrt{3}$]**

(b) Hence show that the area of the trapezium $= \frac{1}{2}(48 + 25\sqrt{3})\ cm^2$

27. (a) Draw a graph of $y = \sin\theta$ for values of from 0 to 420^0 using a scale of 2 *cm* : 300 on the *x*- axis and 4 *cm* : 1 unit on the *y* – axis.

    (b) Use your graph to solve (i) $\sin\theta = {}^-0.7$ (ii) $\sin\theta = 0.5$

    (c) Given that $\text{Cos}\theta = \frac{12}{13}$ for $270^0 \leq \theta \leq 360^0$, find without using tables or calculators the values of: (a) $\text{Sin}\theta$ (b) $\tan\theta$

28. A ladder 10 m long leans against a wall and makes an angle of 63^0 with the horizontal.

    (i) How high up the wall does the ladder reach? **[Ans: 8.91 *m*]**

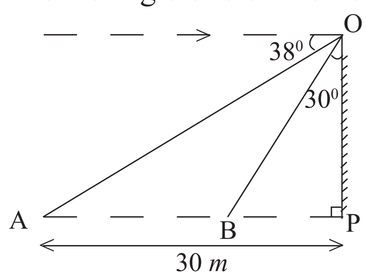
    (ii) How far from the wall is the foot of the ladder? **[Ans: 4.54 *m*]**

29. A vertical post costs a shadow 6 m long on the ground. The height of the post is 15 m. Find the angle of elevation of the sun, correct to the nearest degree. **[Ans: 68^0]**

30. Nairobi fire Brigade has a 10m ladder. The greatest angle at which it can be held against a vertical wall is 750 with the horizontal. What is the maximum vertical height to which it can reach? How far from the wall will it be placed? **[Ans: 9.66 *m*, 2.59 *m*]**

31. From the top of a cliff PO, a man observes two boats A and B.

    (i) What is the angle of elevation of the top O from boat A, boat B? **[Ans: 38^0]**

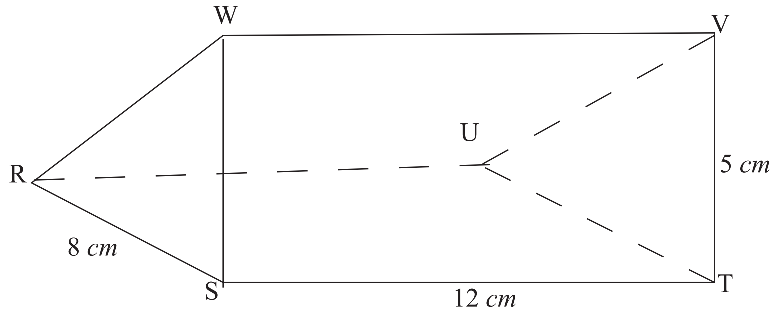


    (ii) What is the angle of depression from O of the nearer boat B and of the farther boat A? **[Ans: 60^0, 38^0]**

MEASURATION

TRIAL QUESTIONS

1. The figure below represents a triangular prism with isosceles ends. Given that RW = WS = VU = VT = 5 *cm*, RS = 8 *cm* and ST = 12 *cm*.



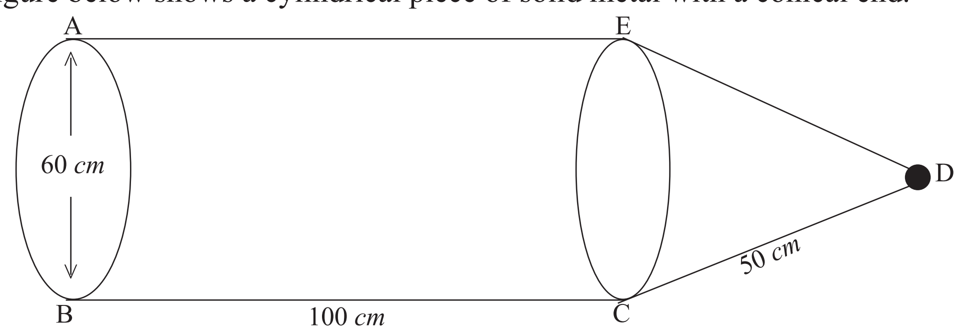
Calculate;

(a) the volume of the prism

(b) the total surface area of the prism

(c) the cost of repainting the prism if a painter charges shs 500 per m^2.

2. A wooden box is 2 *m* long, 0.5 *m* wide and 3 *m* high. Find the total surface area of the box.
3. The figure below shows a cylindrical piece of solid metal with a conical end.



Given that the base diameter AB = 60 *cm*, height of the cylindrical part BC = 100 *cm* and the slant height of the conical part CD = 50 *cm*, calculate:

(a) the volume of the solid

(b) the total surface area of the solid. (Use $\pi = 3.14$)

4. (a) A student had his rectangular photograph of dimensions 30 *cm* by 20 *cm* framed with a uniform border. If the area of the border is 216 cm^2, how wide is the border?

(b) A cone has a radius of 7 *cm* and a vertical height of 30 *cm*. Find

(i) Its volume, $\left(\text{Use } \pi = \frac{22}{7}\right)$.

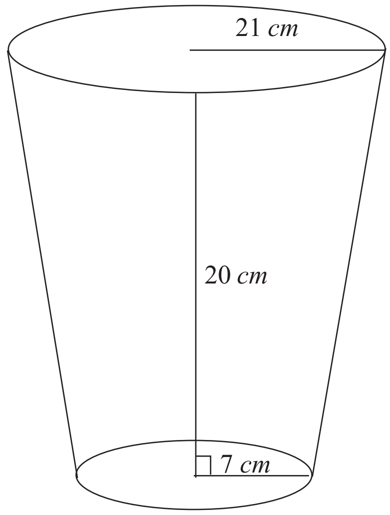
(ii) the volume of another similar bigger cone which has a linear scale factor of 2.

5. A bucket inform of a frustum of a cone has a height of 20 *cm* and radii of 12 *cm* at the top and 8 *cm* at the bottom. If the bucket is open at the top,

(a) Calculate its volume

(b) Surface are of the bucket

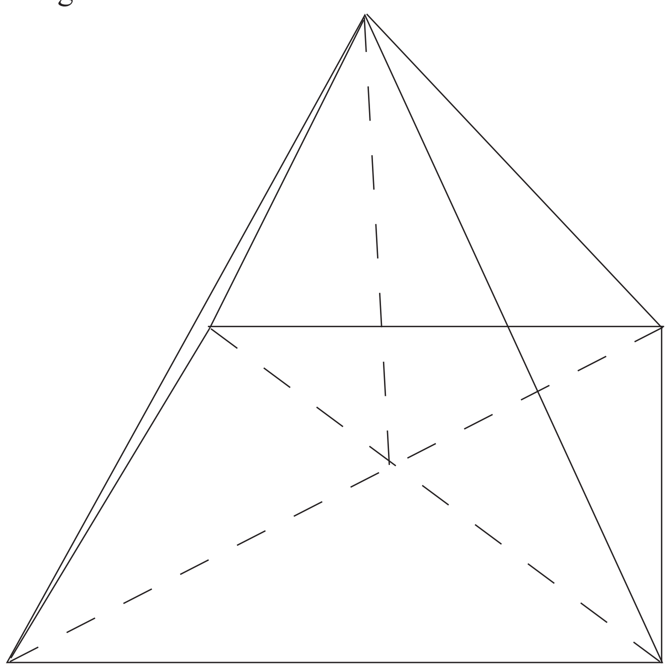
6. The figure below shows a wooden frustrum of a cone. The radius of the base is 7 *cm* and that of the top is 21 *cm*. The frustrum is 20 *cm* high.



(a) Calculate the volume of the bucket, in *cm*³.

(b) If all the water in the bucket is emptied into a cylinder of base radius 14 *cm*, calculate the height of the cylinder, correct to 4 decimal places.

7. The figure below shows a wooden pyramid ABCDV on a square base ABCD of side 40 *cm*. The pyramid is 21 *cm* high as shown.

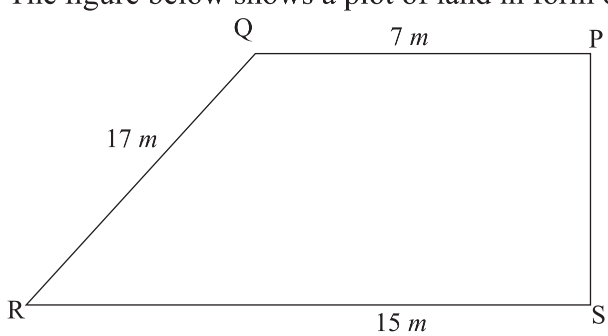


Calculate its;

(i).volume in cm^3.

(ii).total surface area in m^2.

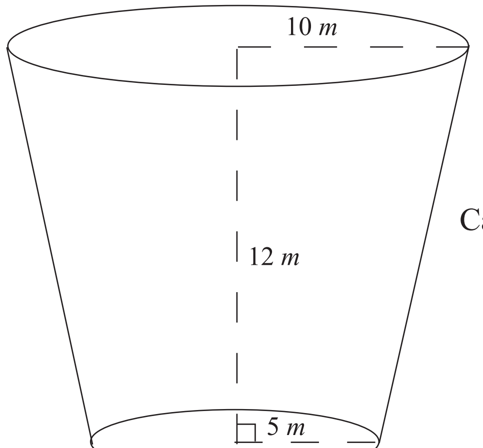
8. (a) The figure below shows a plot of land in form of a trapezium, PQRS.



Calculate; (i) length   hence find its perimeter

(ii) area of the plot in m^2.

(b) The figure below shows a bucket in form of a frustrum of a cone with top radius 10 *m* and bottom radius 5 *m*. The bucket is 12 *m* high



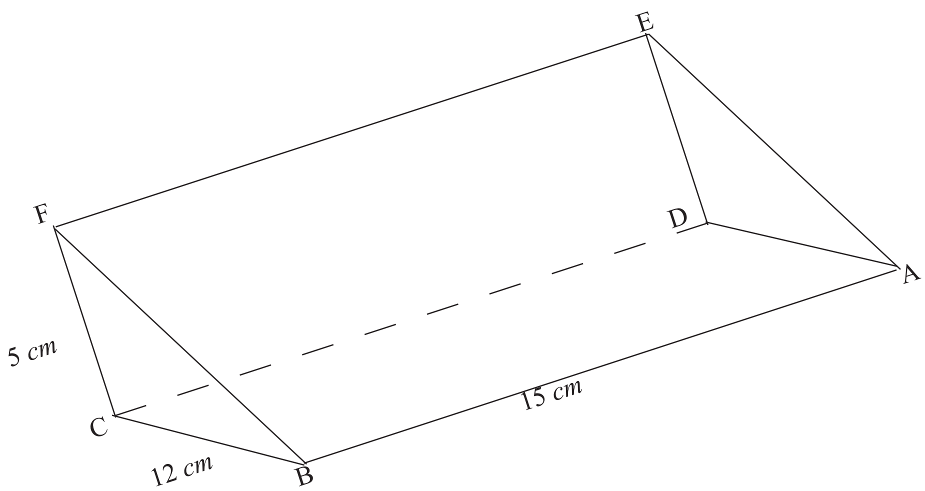
Calculate its volume in m^3. (Take p = $\frac{22}{7}$)

9. A solid in the shape of a frustrum of a cone has radii of 7 *cm* and 14 *cm*. if the original cone had a height of 30 *cm*. calculate;

   (a) Capacity of the solid.

   (b) Its surface area when both ends are closed.

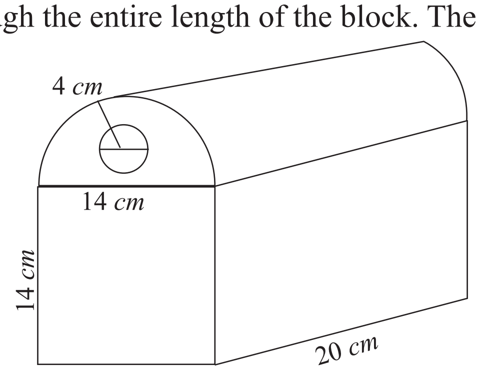
10. The diagram below shows a triangular prism in which AB = 15 cm, BC = 12 *cm*, CF = 5 *cm* and angle BCF = 90^0.



Calculate the; (a) Volume and capacity of the prism.

(b) Total surface area of the prism

11. The diagram below represents a mental block with a uniform cross section. The uniform cross-section consists of a semi-circle of diameter 14 *cm* mounted on a square. A circular hole of diameter 4 *cm* runs through the entire length of the block. The length of the block is 20 *cm*.

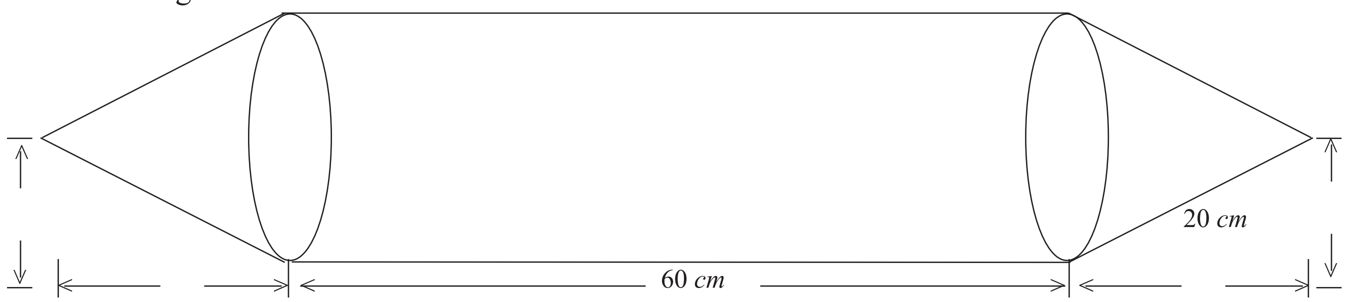


Calculate:

(a) the area in cm^2 of the cross-section

(b) the volume in m^3 of the metal in the block.

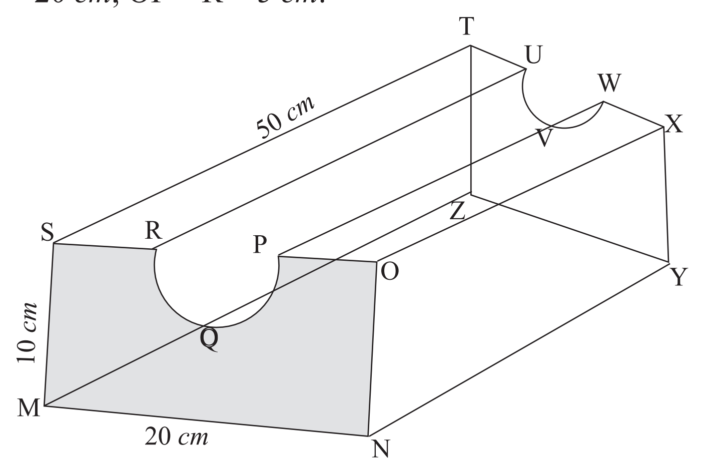
12. The figure below is of a hollow cylinder whose ends is covered by similar cones of radius 14 *cm* and vertical height 20 *cm*.



Calculate; (i) the total surface area of the figure

(ii) the volume of the figure

13. The diagram below shows a solid piece of mental made by drilling a semicircular groove of uniform cross-section PQR in a rectangular block. The radius of the semicircular grove is 5 *cm*. ST = 50 *cm*, SM = 10 *cm*, MN = 20 *cm*, OP = R = 5 *cm*.

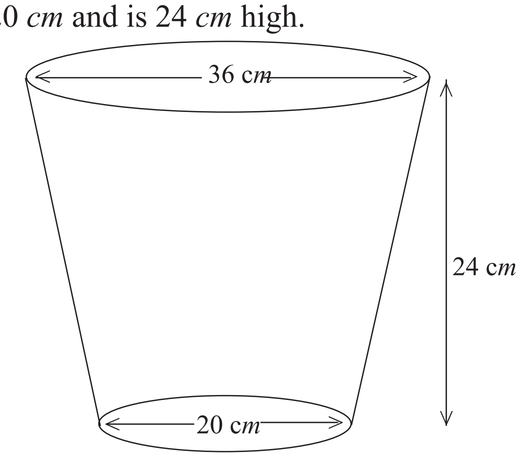


Calculate correct to 3 significant figures the;

(a) area of thr shaded cross-section of the piece of metal.

(b) volume of the metal.

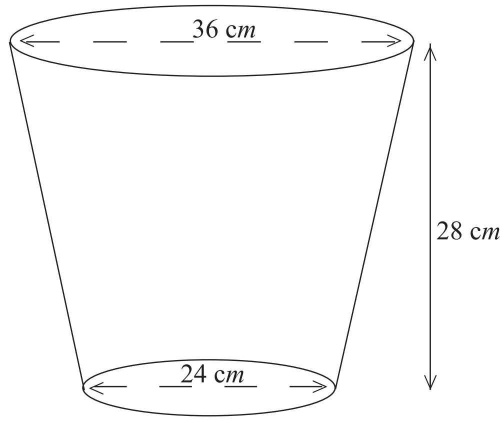
14. (a)The diagram below shows a pail formed from a cone with a top diameter of 36 *cm*, bottom diameter of 20 *cm* and is 24 *cm* high.



(i) Calculate the volume of the pail.

(ii) If this pail is filled with water and the water is later poured to full capacity into a cylinder of base radius 10 *cm*, determine the height of the cylinder. (Take $\pi = 3.142$).

(b) The diagram below shows a bucket in form of a frustrum of a cone with diameters 36 *cm* and 24 *cm* respectively. The bucket is full of water.



(i) Determine the volume of the bucket, in cm^3

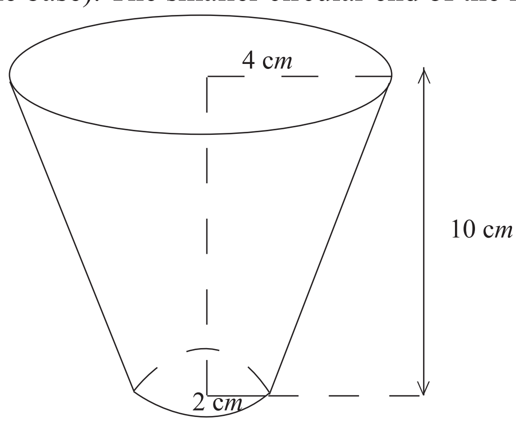
(ii) If the water is poured into a hemisphere, it fills it. Find the radius of the hemisphere. (Use $\pi = \frac{22}{7}$)

15. (a) A hollow plastic cone has a base radius of 4 *cm* and a vertical height of 20 *cm*. Calculate:

(i) the slant height.

(ii) the volume of air inside the cone. [volume = $\frac{1}{3}\pi r^2 h$, $\pi = 3.14$]

(b) The cone described in part (a) above is cut half-way between the top and the base (the cut is parallel to the base). The smaller circular end of the larger piece is then closed to make a drinking cup as below.

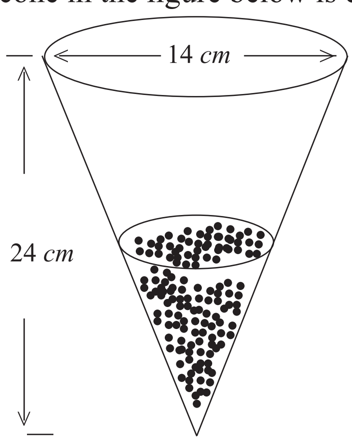


Calculate;

(i) the area of the curved surface of the cup

(ii) the capacity of the cup in millilitres.

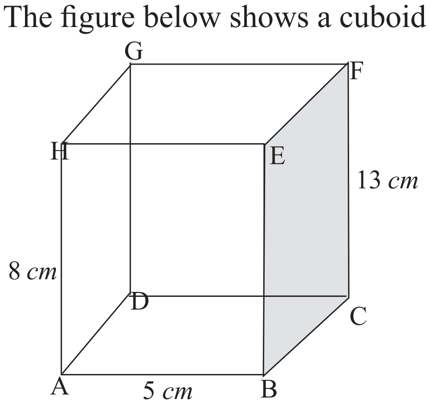
16. The cone in the figure below is exactly full of water by volume.



(a) How deep is the water in the cone?

(b) If the water in the cone is drawn out at a rate of 10 cm^3 per second, find the time taken to empty the cone? (Take $\pi = 3.14$)

17. The figure below shows a cuboid ABCDEFGH with the dimensions as shown.



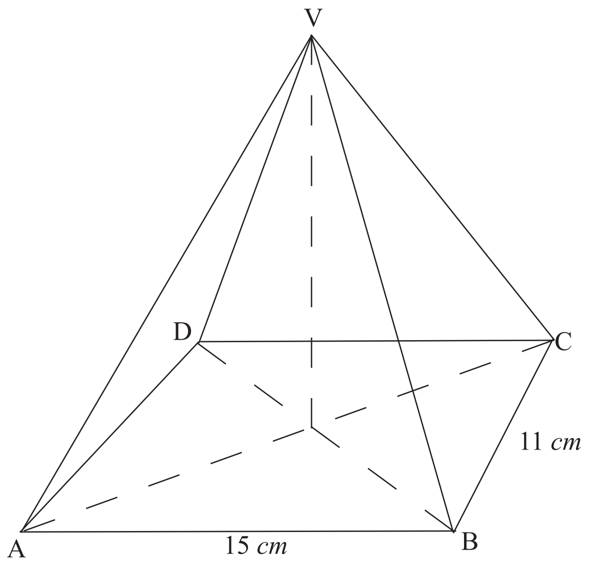
Calculate the;

(i) length BF and BG.

(ii) angle between the line BG and the base ABCD.

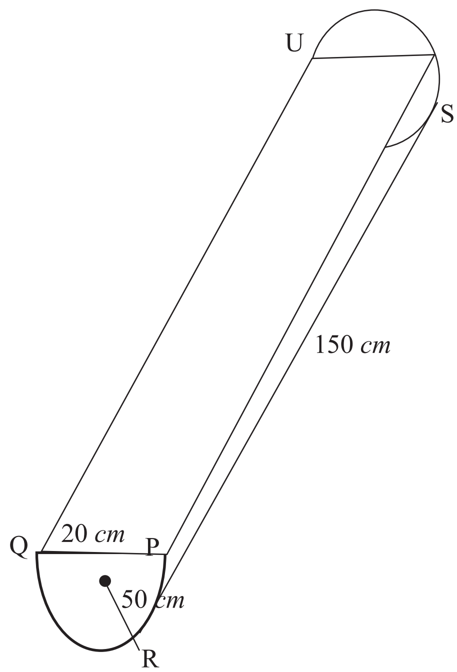
(iii) angle between the plane BDF and the base ABCD.

18. The figure below represents a right pyramid with vertex, V and a rectangular base ABCD. VA = VB = VC = VD = 20 *cm*, AB = 15 *cm* and BC = 11 *cm*. M is the mid point of BC. Find the;



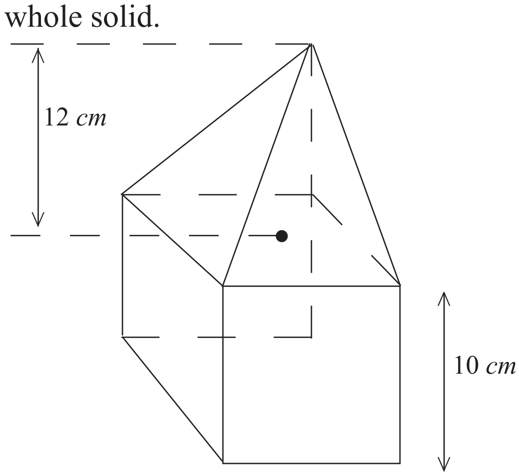
(a) height V is above the base
(b) angle of elevation of V from A
(c) angle between planes VBC and ABCD.

19. Diagram below shows a hollow right cylinder PQRSTV of neglible thickness. Part of which has been cut off as shown below. If the radius of the circular end is 50 *cm* RS = 150 cm and PQ = TU = 20 *cm*.

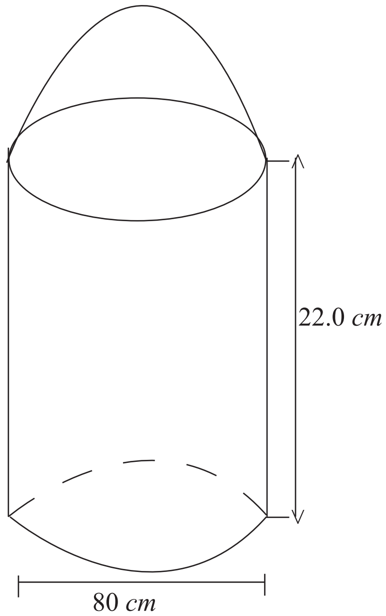


Find;
(i) the area of the cross section PQR
(ii) How much water (in litres) would fill this container
(Use $\pi = 3.14$

20. The figure below shows a cube of side 10 cm surmounted by a pyramid of vertical height 12 cm. Find the volume of the whole solid.



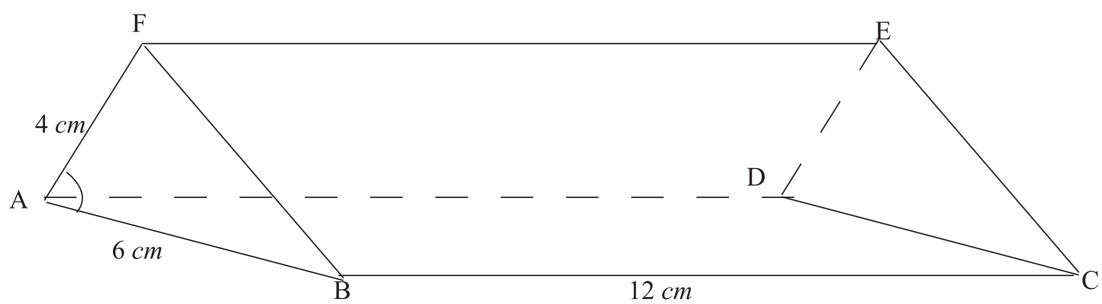
21. A flask is made from a cylinder of diameter 80 *cm* and height 22.0 *cm* topped by a hemisphere.



Find the total volume of the flask

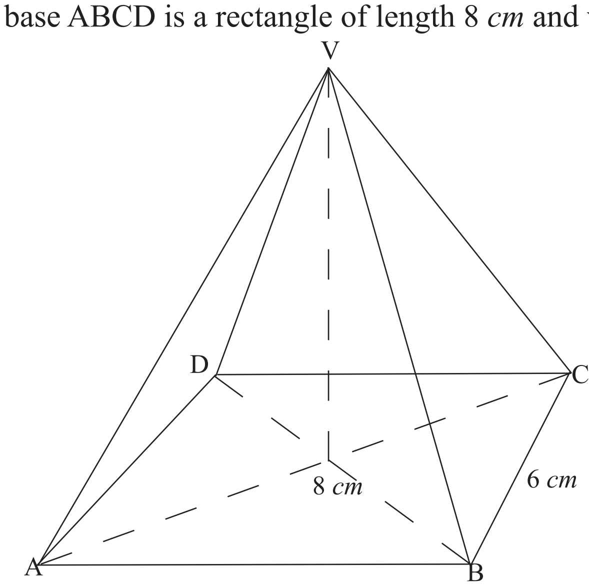
22. VABCD is a pyramid with a square base ABCD of side 5 *cm*. V is its vertex vertically above the base such that VA = VB = VC = VD = 8 *cm* and O is the centre of the base. Calculate the;
    (a) height OV of the pyramid (Correct to 2d.p)
    (b) angle VA makes with the base ABCD
    (c) angle between plane VDC and the base ABCD
    (d) volume of the pyramid (correct to four significant figures).

23. (a) The figure below shows a triangular prism of uniform cross-section ABF in which AF = 4 *cm*, AB = 6 *cm* and BC = 12 *cm*.



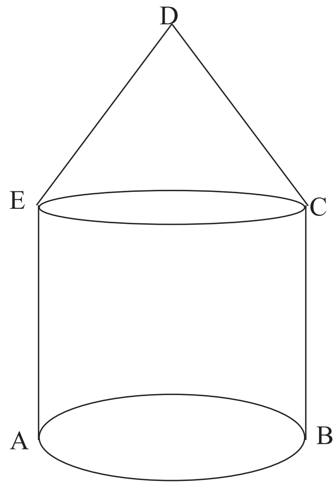
If angle BAF = 30^0, calculate the volume of the prism.

24. The figure below shows a right pyramid with vertex V and the edges VA = VB = VC = VD = 13 *cm*. The base ABCD is a rectangle of length 8 *cm* and width 6 *cm*.



Calculate;
(i) the height of the pyramid.
(ii) the volume of the pyramid in cm^3.
(iii) the total surface area of the pyramid.

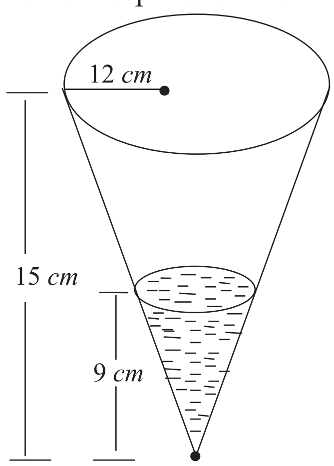
25. The diagram below shows a giant water tank ABCDE which is made in the shape of a right cone mounted on a cylinder. The radius of the cylinder is 6 *m*. The slate length of the cone ED = CD = 7.5 *m* and the height of the cylinder BC = AE = 10 *m*.



(a) If the water tank has to be painted on the outer surface, calculate the surface area to be painted in m^2. (Take $\pi = 3.142$).

(b) Calculate the volume of water needed to fill the tank in m^3. (Take $\pi = 3.142$).

26. The diagram below shows a right cone – shaped flask of base radius 12 *cm* and a depth of 15cm containing water to a depth of 9 *cm*.

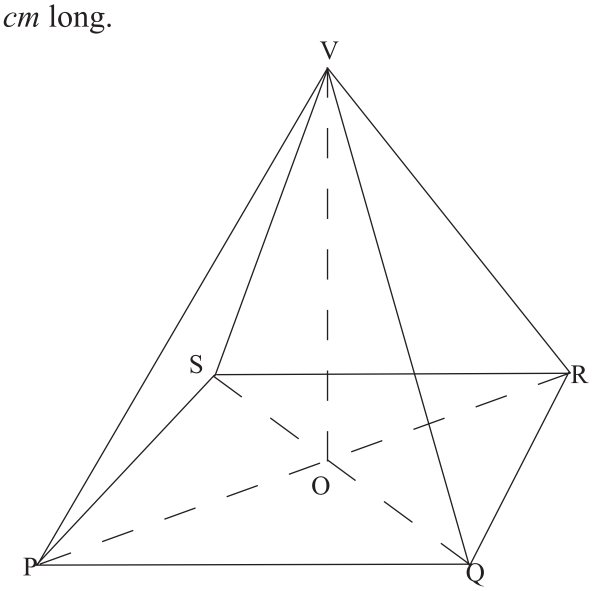


(a) Calculate the radius of the surface of water

(b) If the water is poured in a cylinder of radius 4cm and it fills the cylinder, determine the height of the cylinder.

(c).If the water is poured in a cube to the brim, find the length of the side of the cube. (take $\pi = 3.14$ and volume of cone $= \frac{1}{3}\pi r^2 h$)

27. VPQRS is a right pyramid with a rectangular base. PQ = 48 *cm*, QR = 36 *cm* and each slant edge is 50 *cm* long.



Calculate the;

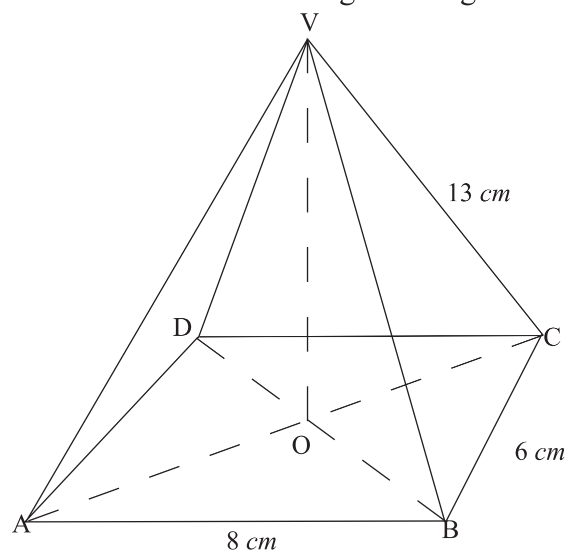
(a) the height of the pyramid.

(b) angle between the slanting edge and the base

(c) angle between the plane VQR and the base.

(d) angle between planes VQR and VPS

28. The diagram below shows a right – pyramid with vertex V and the edges VA = VB = VC = VD = 13 *cm*. The base ABCD is a rectangle of length 8 *cm* and width 6 *cm*.



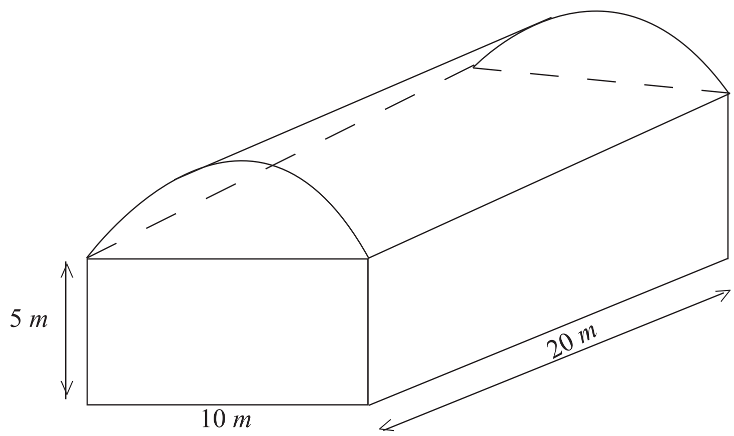
Calculate the;

(a) vertical height OV

(b) volume of the pyramid

(c) total surface area of the pyramid

29. The figure below shows a building which consists of a cuboid 20 *m* long, 10 *m* wide and 5 *m* high surmounted by half a cylinder, diameter 10 *m* and length 20 *m*.

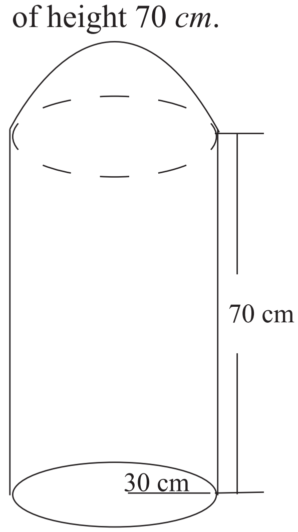


Calculate

(i) the volume of the building

(ii) the total area of the end, walls and roof

30. Figure below shows a hot water tank made by joining a hemisphere of radius 30 *cm* to an open cylinder of height 70 *cm*.



(a) Calculate the total surface area of the tank.

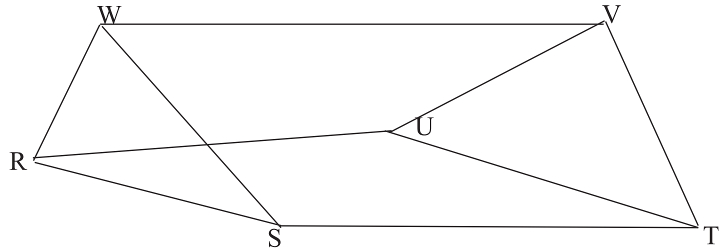
(b) The tank is full of water

(i) Calculate the capacity of the tank in litres

(ii) The water drains from the tank at a rate of 3 litres per second. Calculate the time, in minutes taken to empty the tank.

(surface area of a sphere = $4\pi r^2$,volume of a sphere $\frac{4}{3}\pi r^3$)

31. The figure below is a triangular tent with isosceles ends. Given that RW = WS = VU = VT = 5 *m*, RS = 8 *m* and ST = 12 *m*.

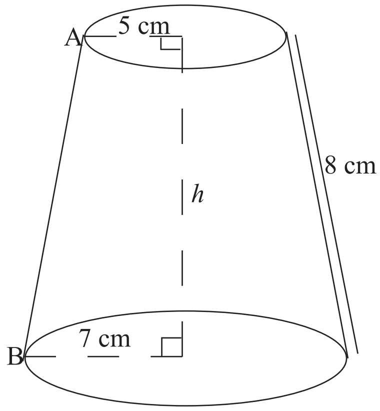


Calculate: (a) the volume of the tent.

(b) the total surface area of the tent.

(c) the cost of repainting this tent if a painter charge sh 500 per m^2.

32. The figure below shows a lampshade in the form of a conical frustum.

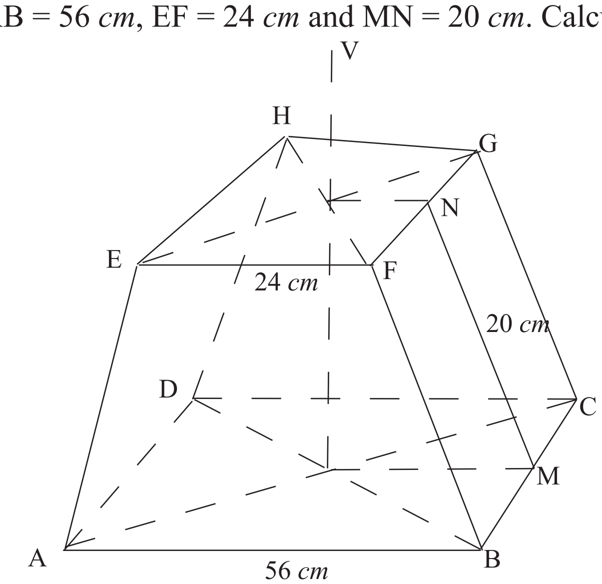


The top and bottom radii are 5 *cm* and 7 *cm* respectively. The slant height AB is 8 *cm*.

(a) Calculate the volume of the lamp shade to 4 significant figures.

(b) Find the vertical height of the lampshade. (Take $\pi = 3.142$)

33. The diagram below shows a solid block of wood ABCDEFGH which is part of a right pyramid on a square base ABCD with its vertex at V. Given that M and N are the mid-points of BC and FG respectively, AB = 56 *cm*, EF = 24 *cm* and MN = 20 *cm*. Calculate the:

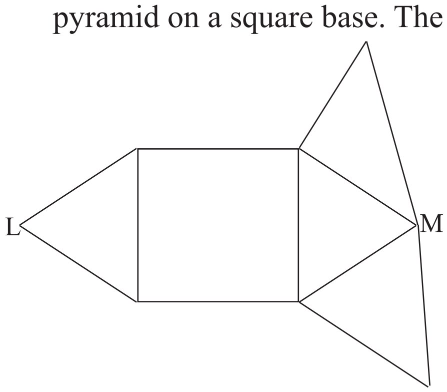


(a) height ST of the block of wood

(b) total surface area of the block of wood

(c) volume of the block of wood

34. The figure shows a square of side 6 *cm* and four congruent isosceles triangles. It represents the net of a pyramid on a square base. The distance LM is 20 *cm*. Calculate;



(a) The total surface area of the pyramid

(b) The perpendicular height of the pyramid when the net is folded.

(c) The angle of inclination of a triangular face to the base of the pyramid.

35. Two similar rectangular cartons have bases 15 *cm* long and 30 *cm* long. The smaller has a volume of 1200 cm^3. Find the volume of the larger.

36. The base of a triangle is 7.5 *m* and an area of 81 m^2. Another similar triangle has a base 2.5 *m*. What is the area of the smaller triangle?

37. A map is drawn to a scale 1:250,000 Find the actual area in km^2 of a lake which on the map has an area of 2 cm^2.

38. A liquid is filled in two similarly shaped test tubes. The smaller one can contain 25 cm^3 and the larger one 200 cm^3. If the smaller test tube is 4 *cm* long, How long is the larger one?

39. Two solids are of similar shape with linear scale factor 3. If one has a surface area of 36 cm^2 and a volume of 12 cm^3, what are the surface area and volume of the other?

40. The surface area of two similar spheres are in the ratio 4:9. If the volume of the larger sphere is 135 cm^3, find the volume of the smaller one.

41.A cylindrical tank 7.5 *m* high has a capacity of 540 litres. The area of its base is 180 m^2. A similar tank is 5 *m* high. Find the base area and the capacity of this small tank.

42. A notice board 50 *cm* high has an area of 1000 cm^2. Another similar shaped notice board is twice as high . Find :

(i) the area scale factor

(ii) the area of the larger notice board.

VECTORS

RECALL

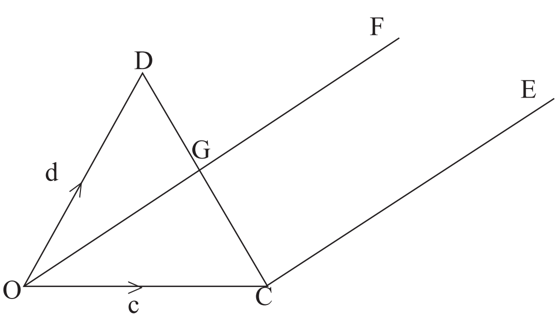
- ◊ A vector can be represented as a column vector, $\begin{pmatrix} x \\ y \end{pmatrix}$ where x denotes the horizontal movement and y denotes vertical movement.
- ◊ To add or subtract vectors, we add or subtract corresponding components.

  For example;

  $$\begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ ^{-}1 \end{pmatrix} = \begin{pmatrix} 1+3 \\ ^{-}1+2 \end{pmatrix} = \begin{pmatrix} 4 \\ ^{-}1 \end{pmatrix}$$
- ◊ To multiply a vector by a scalar, multiply each component by a scalar. For example;

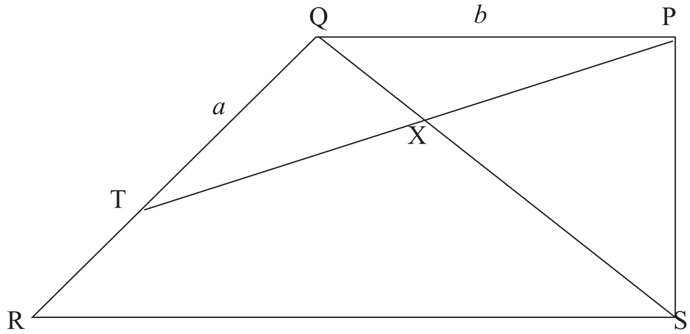
  $$2\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \times 2 \\ 2 \times 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$$
- ◊ Length of a vector is called a magnitude and is calculated as: $|p| = \sqrt{x^2 + y^2}$
- ◊ Reverse direction in vectors is denoted with a negative i.e OA = $^{-}$AO
- ◊ If one vector is a scalar multiple of the other, then the two vectors are parallel ie **PQ = γDR**
- ◊ Collinear points are points that lie on a straight line. They have a common point as parallel.
- ◊ Consider points A, B and C are on a straight line,then $\frac{AB}{BC}$ = **μ or AB= μBC where μ is a scalar**

TRIAL QUESTIONS

1. Two points P(5, 2) and Q (2, 4) are in a plane. Find the
   (a) Co-ordinates of M the midpoint of PQ
   (b) |0M|, where O is the origin
2. (a) If AB = $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and OB = $\begin{pmatrix} 5 \\ 1 \end{pmatrix}$, find the coordinates of point A.
   (b) The coordinates of R and S are (2, 7) and (14, 2) respectively. Find:
   (i) RS (ii) the magnitude of RS.
3. The position vectors of points R and T are r = $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$ 1 and t = $\begin{pmatrix} 4 \\ -7 \end{pmatrix}$ respectively. Find the magnitude of RT.
4. The co-ordinates of A and B are ($^{-}$6, 15) and (4, 5) respectively, x is a point on AB such that AX:XB = 1:4.
   Find (i) AB (ii) OX
5. Given that Q϶PR such that PQ: :QR = 3 : 2, find the co-ordinates of Q if P is (2,5) and R is (7, $^{-}$5)
6. 

   In the diagram above Oc = c , OD = d, 7DG = 4DC and OG: GF = 5:2
   (a) Find in terms of c and d
   (i) DG (ii) OG (iii) OF
   (b) If OE = $\frac{1}{5}$ (12c - d) show that D, F and E are collinear.
7. Given that OQ = $\begin{pmatrix} ^{-}2 \\ 10 \end{pmatrix}$ and OP = $\begin{pmatrix} 10 \\ -4 \end{pmatrix}$
   (a) Determine the column vector for PQ
   (b) Hence find the length of vector PQ

8.



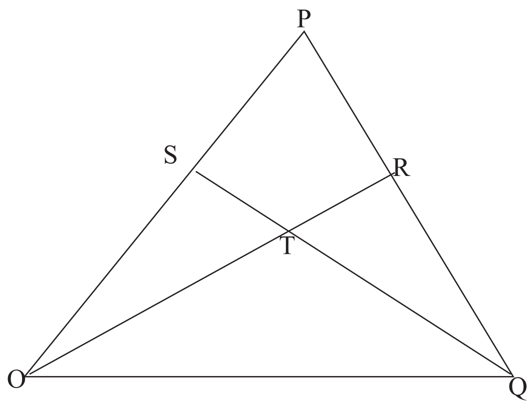
In the figure above vectors QT = a, QP = b, QR = $3a$, and RS = $2b$.

(a) Express the following vectors in terms of a and b.
(i) PT (ii) QS

(b) If PX = kPT, express QX in terms of a, b and k where k is a scalar.

(c) If QX = tQS, find the values of k and t

9.



In the figure OPQ is a triangle in which OS = $\frac{3}{4}$ OP and PR : RQ = 2 : 1. Lines OR and SQ meet at T.

(a) Given that OP = p and OQ = q, express the following vectors in terms of p and q;

(i) $\overrightarrow{PQ}$ (ii) $\overrightarrow{OR}$ (iii) $\overrightarrow{SQ}$

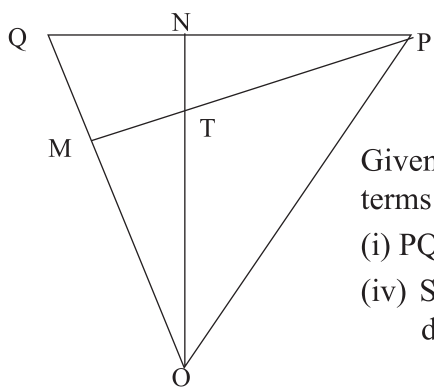
(b) If ST = mSQ and OT = nOR, determine the values of m and n.

10. (a) Given that a = $\begin{pmatrix} ^-2 \\ -9 \end{pmatrix}$, b = $\begin{pmatrix} 7 \\ 2 \end{pmatrix}$ and $m = a + 2b$ find the magnitude of m.

(b) Given that S(-2 6) and T(3 3) are two points, find the coordinates of R if $\overrightarrow{OR} = 4\overrightarrow{OS} + \frac{1}{3}\overrightarrow{OT}$ and O is the origin.

11. (a) Given that $\overrightarrow{OA} = \begin{pmatrix} 12 \\ 16 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$, calculate the length of AB.

(b) The diagram shows triangle OPQ in which QN : NP = 1 : 2, OT : TN = 3 : 2 and M is the mid point of $\overrightarrow{OQ}$.



Given that OP = p and OQ = q express the following vectors in terms of p and q.

(i) PQ (ii) ON (iii) PT

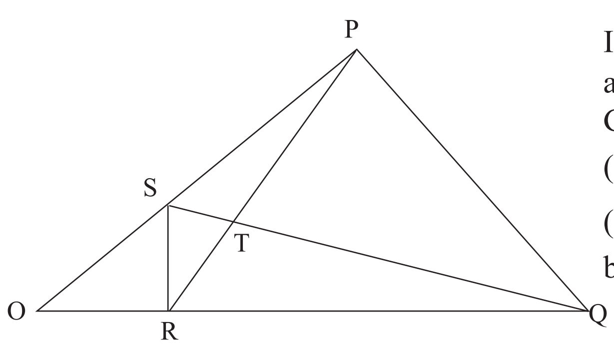
(iv) Show that the point P, T and M are collinear and hence determine the ratio MT : TP.

12. (a) Given that $a = \begin{pmatrix} 12 \\ 16 \end{pmatrix}$ and $b = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$. Calculate; $|a + b|$.

(b) Given that $a = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$, $b = \begin{pmatrix} ^-2 \\ 1 \end{pmatrix}$ and $c = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$, find the magnitude of $a + 2b + 3c$.

(c) Given that $a = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $b = \begin{pmatrix} ^-1 \\ 0 \end{pmatrix}$ and $c = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$. Evaluate $\left|2a - 2b + \frac{3}{8}c\right|$

13.



In the above figure OPQ in a triangle in which OS = $\frac{1}{3}$ OP and OR = $\frac{1}{3}$ OQ. T is a point on QS such that QT = $\frac{3}{4}$ QS. Given that OP = p and OQ = q.

(a) Express the following vectors in terms of p and q

(i) $\overrightarrow{SR}$ (ii) $\overrightarrow{QS}$ (iii) $\overrightarrow{PT}$ (iv) $\overrightarrow{TR}$.

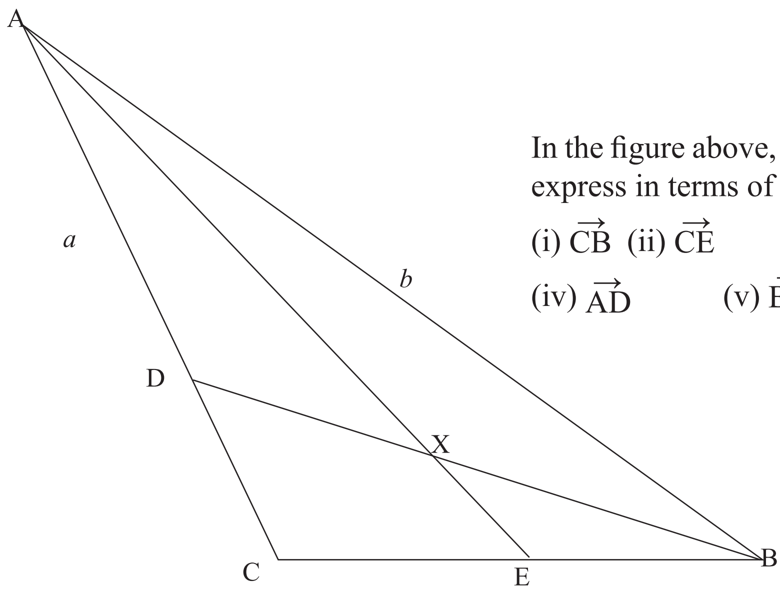
b) Find the ratio of PR to TR.

14. (a) Given the vectors $a = \begin{pmatrix} ^-2 \\ 1 \end{pmatrix}$, $b = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$ and $c = \begin{pmatrix} 10 \\ 14 \end{pmatrix}$, find the values of the constants p and q such that c = $pa + qb$ without using tables or calculator.

(b) M is the point ($^-6$, 8). If OM = a - 2b and $a = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$, find;

(i) the value of vector b (ii) the magnitude of b

15.



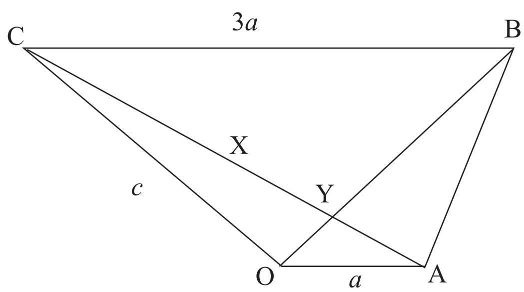
In the figure above, $\overrightarrow{AB} = b$, $\overrightarrow{AC} = c$, 2AD = 3DC, 3CE = CB, express in terms of b and c.

(i) $\overrightarrow{CB}$ (ii) $\overrightarrow{CE}$ (iii) $\overrightarrow{AE}$

(iv) $\overrightarrow{AD}$ (v) $\overrightarrow{BD}$ (vi) $\overrightarrow{DE}$

16. Given that $\overrightarrow{OL} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\overrightarrow{OM} = \begin{pmatrix} 7 \\ 7 \end{pmatrix}$ are position vectors of points L and M respectively. Find the length of $\overrightarrow{LM}$.

17.



The diagram above shows a trapezium OABC. $\overrightarrow{OA} = a$, $\overrightarrow{OC} = c$ and $\overrightarrow{CB} = 3a$. X and Y are points on $\overrightarrow{AC}$ such that AX : XC = 1 : 2 and AY : YC = 1 : 3.

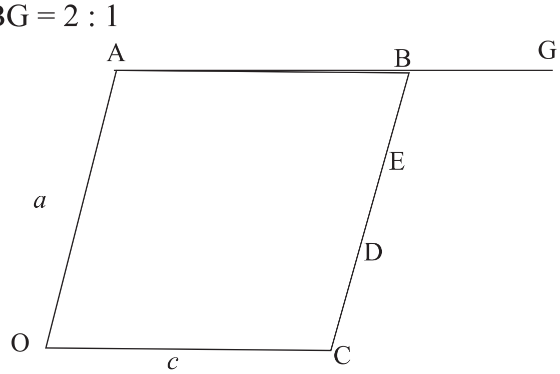
(a) Express the following vectors in terms of a and c.

(i) $\overrightarrow{AC}$ (ii) $\overrightarrow{OY}$ (iii) $\overrightarrow{OX}$ (iv) $\overrightarrow{AB}$ (v) $\overrightarrow{AY}$

(b) Hence, show that O, Y and B are collinear.

(c) OX is produced and cuts CB at Z. Find the ratio OX : XZ

18. In the diagram below, OABC is a parallelogram with OA = a, OC = c, CD = DE, DE : CB = 1:3 and AB : BG = 2 : 1



(a) Find in terms of a and c the vectors DA and DG.

(b) Show that the points O, E and G are collinear.

(c) Find the ratio OE : OG.

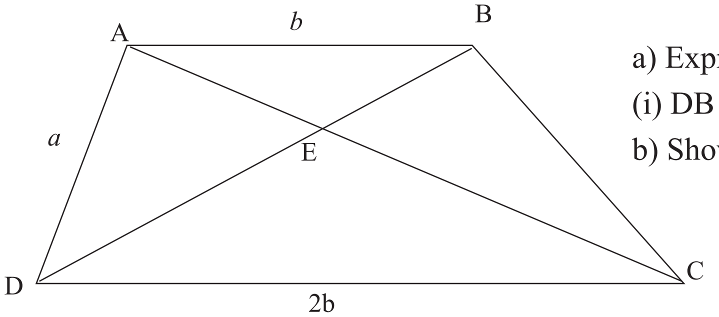
19. In a triangle ABC, L and M are points on AB and BC respectively such that AL= $\frac{2}{3}$AB and BM = $\frac{1}{3}$BC. X is a point on AM such that AX = $\frac{3}{4}$AM. Given that AB = b and AC = c, express the following vectors in terms of b and c.

(i) AM (ii) CL (iii) CX

(b) Show that C, X and L are collinear points.

(c) State the ratio CL: CX.

20. The diagram below shows a quadrilateral ABCD with DE = 2EB, $\overrightarrow{AB} = b$, $\overrightarrow{DC} = 2b$ and $\overrightarrow{DA} = a$.

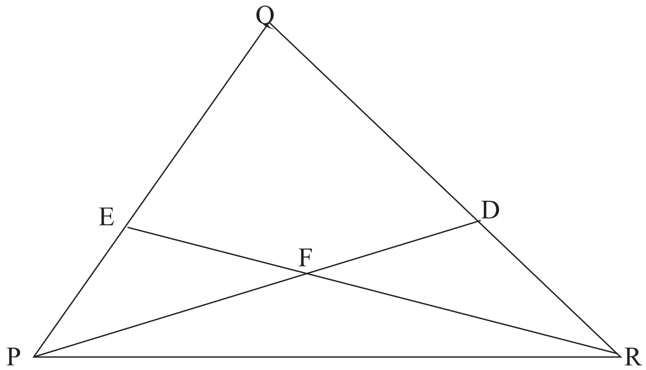


a) Express in terms of a and b the vectors;

(i) DB (ii) BE (iii) AE (iv) AC

b) Show that the points C, E and A are collinear.

21. In triangle PQR, PR $= p$ and PQ $= q$. E divides PQ in the ratio 1: 3 and D divides QR in the ratio 5 : 2. DP meets RE at F.



(a) Express the following vectors in terms of p and q.

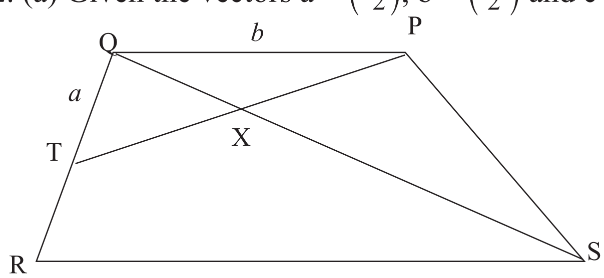
(i) PD (ii) RE

(b) If PF = hPD and $\overrightarrow{RF}$ = kPE where h and k are scalars, by expressing in two different ways find the values of h and k.

(c) Hence find the ratio EF : FR.

22. (a) Given the vectors a = $\begin{pmatrix}3\\-2\end{pmatrix}$, b = $\begin{pmatrix}^{-}1\\2\end{pmatrix}$ and c = $\begin{pmatrix}^{-}4\\2\end{pmatrix}$, find the magnitude of $3a - 4b + \frac{1}{2}c$.

b)



In the figure above, vectors QT = a and QP = b.

(i) Express vector $\overrightarrow{PT}$ in terms of a and b.

(ii) If PX = kPT, express $\overrightarrow{QX}$ in terms of a, b and k where is a scalar.

(iii) If $\overrightarrow{QR} = 3a$ and $\overrightarrow{RS} = 2b$, express $\overrightarrow{QS}$ in terms of a and b.

iv) If $\overrightarrow{QX} = h\overrightarrow{QS}$, find the values of k and h.

23. Given that; $\overrightarrow{OA} = \begin{pmatrix}6\\-3\end{pmatrix}$ and $\overrightarrow{AB} = \begin{pmatrix}4\\9\end{pmatrix}$; find; (i) $\overrightarrow{OB}$ (ii) $|\overrightarrow{OB}|$

24. OABC is a quadrilateral. P, Q, R are points on OA , OB , OC respectively such that $\overrightarrow{OA} = 3\overrightarrow{OP}$, $\overrightarrow{OB} = 5\overrightarrow{OQ}$ and $\overrightarrow{OC} = 2\overrightarrow{OR}$.

(a) Given that OP = p, OQ = q and OR = r, Find in terms of p, q and r, the vectors:

(i) AB (ii) BC (iii) CA

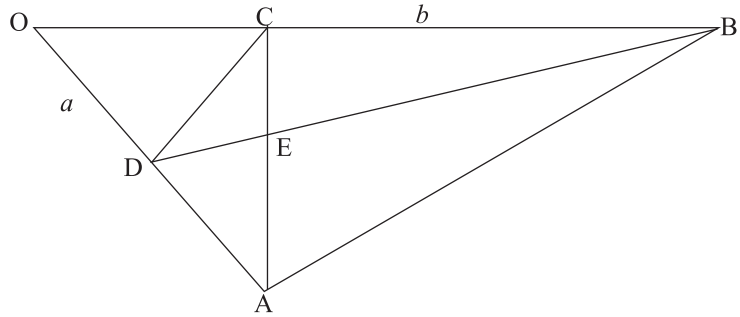
b) If OABC is a parallelogram, show that;

$3p - 5q + 2r$ hence express $\overrightarrow{PQ}$ and $\overrightarrow{QR}$ in terms of p and q only.

(c) Show that P, Q and R are collinear

(d) Find the ratio PQ : QR

25. In the figure below, OAB is a triangle such that 30D = OA, 3OC = OB. E is a point on BD such that 4BE = 3BD.



(a) Given that OA = a, OB = b, express the following vectors in terms of a and b.

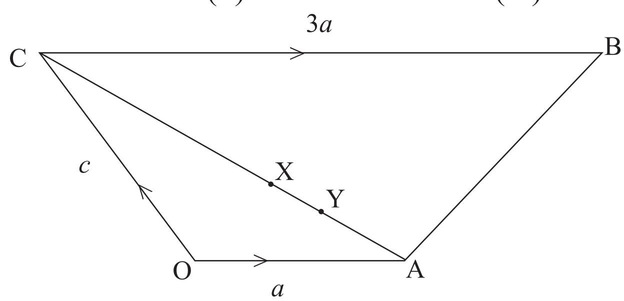
(i) DC (ii) BD (iii) AE (iv) EC

(b) Show that the points A, E and C are in a straight line.

26. The diagram below shows a trapezium OABC. OA = a, OC = c and CB = $3a$. X and Y are points on AC such that AX : XC = 1: 2 and AY : YC = 1: 3

Give the following vectors in terms of ***a*** and ***c***

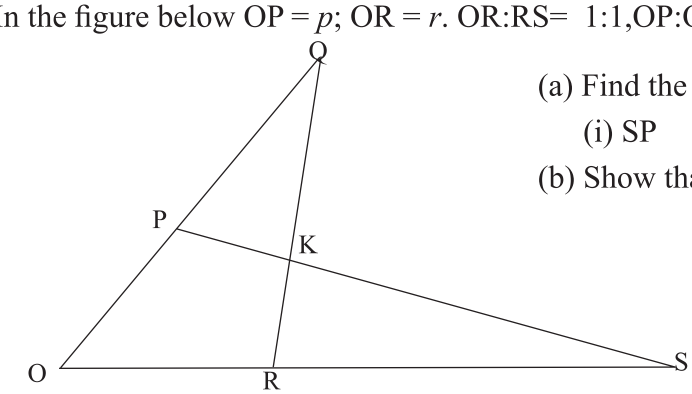
(i) AC (ii) OY (iii) OX (iv) AB (v) AY



Hence, show that O, Y and B are collinear.

OX is produced and cuts CB at Z. Find the ratio OX : XZ

27. In the figure below OP = p; OR = r. OR:RS= 1:1,OP:OQ =2:3 and 2QK = QR

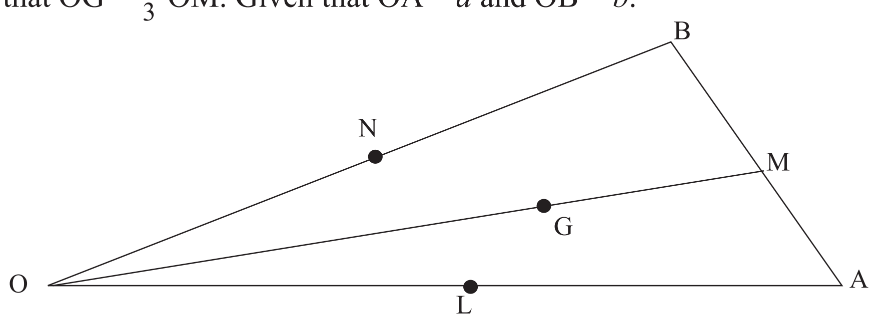


(a) Find the following vectors in terms of p and r.

(i) SP (ii) OK (iii) PK

(b) Show that the points P, K and S are collinear.

28. In the figure below L, M and N are the mid points of OA, AB and OB respectively. G is a point on OM such that OG = $\frac{2}{3}$ OM. Given that OA = a and OB = b.

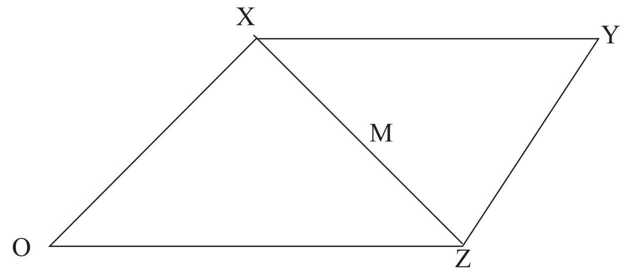


(a)Express in terms of a and b the vectors (i) OM (ii) OG (iii) BG (iv) GL

(b) Show that points B, G and L are collinear

29. Point A has coordinates (3, 2) and AB = $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$. Find the magnitude of 2OA + OB.

30. In the figure below, M is the mid point of XZ. $\overrightarrow{OX} = p + 2q$, $\overrightarrow{OZ} = 7p + 2q$ and $\overrightarrow{OY} = 3kq + np$ where k and n are constants and p and q are vectors.



(a) Express as simply as possible in terms of p and q each of the following vectors.

(i) $\overrightarrow{XZ}$ (ii) $\overrightarrow{XM}$ (iii) $\overrightarrow{OM}$

(b) Express in terms of p, q, k and n as simply as possible.

(c) If Y lies on OM produced and OY:YM = 3:2, find the values of k and n hence express $\overrightarrow{OY}$ in terms of p and q only.

TRANSFORMATION, TRANSLATION, REFLECTION & ENLARGEMENT

REVIEW

Reflection

- ◊ An object and mirror line are required such that an image is found after reflection by the mirror. Or, when we are given the object and it's image, we can find the position of the mirror line.
- ◊ The object and the image are oppositely congruent.
- ◊ The image is of the same size as the object.
- ◊ The line joining the object to it's corresponding image meets the mirror line at 90^0.

Transformation

- ◊ Transformation × Object = Image
- ◊ T × O = I

Transformation and area scale factor

- ◊ Area scale factor = determinant of a matrix of transformation
- ◊ Area scale factor = $\frac{\text{Area of the image}}{\text{Area of the object}}$
- ◊ Area scale factor = (Linear scale factor)^2

Combined or Successive Transformtions

- ◊ Suppose that the triangle P undergoes transformation T_1 to produce P′ followed by a transformation T_2 to produce P" then the single transformation T that maps P to P" is $T_2 \times T_1$.
- ◊ $T = T_1 + T_2$
- ◊ A single matrix that maps P" back to P is the inverse matrix of the successive transformation (T^{-1})

Matrix Transformations

Transformations	Matrix of transformation
Reflection in the x- axis	$\begin{pmatrix} 1 & 0 \\ 0 & ^-1 \end{pmatrix}$
Reflection in the y- axis	$\begin{pmatrix} ^-1 & 0 \\ 0 & 1 \end{pmatrix}$
Reflection in the line $y = x$ or $y - x = 0$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Reflection in the line $y + x = 0$ or $y = {}^-x$	$\begin{pmatrix} 0 & ^-1 \\ ^-1 & 0 \end{pmatrix}$
Positive quarter (90^0) turn about (0, 0)	$\begin{pmatrix} 0 & ^-1 \\ 1 & 0 \end{pmatrix}$
Negative quarter ($^-90^0$) turn about (0, 0)	$\begin{pmatrix} 0 & 1 \\ ^-1 & 0 \end{pmatrix}$
Half turn (+180) about (0, 0)	$\begin{pmatrix} ^-1 & 0 \\ 0 & ^-1 \end{pmatrix}$
Identity	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

- ◊ General matrix for rotation; $T = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$
  Where θ is the given angle.

TRIAL QUESTIONS

1. On graph papers, plot the points A(1, 3) B (3, 3) and C (3, 1). Join the points to form triangle ABC. Find the co-ordinates of A', B' and C' after an enlargement of
   (a) Scale factor 2 with centre of enlargement (0, 0)
   (b) Scale factor $^-1$ with centre of enlargement al (0, 0)
   (c) Scale factor $^-2\frac{1}{2}$ with centre of enlargement al (2, 2)
2. (a) The vertices of triangle RPQ are (0, 3), (1, 5) and (3, 3) respectively. Find the image $R_1P_1Q_1$ of triangle RPQ under a reflection in the line $x + y = 0$. **[Ans: $R_1(^-3, 3)$ $P_1(^-5, ^-1)$ $Q_1(^-3, ^-1)$]**
   (b)The image $R_1P_1Q_1$ is then mapped on to $R_2P_2Q_2$ under a rotation about the origin through 270^0. Find the coordinates of the vertices of triangle $R_2P_2Q_2$. **[Ans: R_2 (3, 3) $P_2(^-1, 5)$ $Q_2(^-1, 3)$]**
   (c) Find a single transformation matrix that will map triangle $R_2P_2Q_2$ back to RPQ
   (d) Describe the transformation in (c) above fully.
3. (a) Plot the points A(1, 0), B(0, $^-2$), C(2, 0) on a graph using a scale of 2 *cm* : 1 unit on both axes.
   (b) Graphically, find the image
      (i) triangle $A_1B_1C_1$ of triangle ABC after an enlargement by scale factor $^+2$ about the origin.
      (ii) triangle $A_2B_2C_2$ of triangle ABC after an enlargement by scale factor $^-2$ about the origin.
   (c) If triangle A_1B1C_1 is the image of triangle $A_2B_2C_2$ under an enlargement, find:
      (i) the centre of enlargement
      (ii) the scale factor.
4. Plot on the same axes the
   (i) vertices P(2, 0), Q (0, 2), R ($^-2$, 0) and S(0, $^-2$)
   (ii) image P′ Q′ R′ S′ of PQRS under the matrix transformation $\begin{pmatrix} 2 & 0 \\ 4 & 2 \end{pmatrix}$
   (iii) Image P"Q"R"S" of P′ Q′ R′ S′ under a reflection in the x–axis. Determine the matrix of this transformation
   (b) Find the matrix M of the transformation which maps PQRS onto P"Q"R"S" **[Ans: $\begin{pmatrix} 2 & 0 \\ ^-4 & ^-2 \end{pmatrix}$]**
5. (a) The image of quadrilateral P(2, $^-1$) Q(0, $^-3$) R(2, $^-4$) S(4, $^-2$) is P_1(1, 2), Q(3, 0), R_1(4,2) and S_1(2, 4) under a certain transformation T_1
   (i) Find the matrix of transformation for T_1 **[Ans: $T_1 = \begin{pmatrix} 0 & ^-1 \\ 1 & 0 \end{pmatrix}$]**
   (ii) Describe the transformation T_1 fully **[Ans: A matrix of reflection in positive quarter turn]**
   (b) Given that quadrilateral $P_1Q_1R_1S_1$ is then mapped onto quadrilateral $P_2Q_2R_2S_2$ after a reflection in the line $y + x = 0$ .Find the coordinates of the vertices of quadrilateral $P_2Q_2R_2S_2$
   [Ans: P2 ($^-2$, $^-1$) Q2 (0, $^-3$) R2 ($^-2$, $^-4$) S2 ($^-4$, $^-2$)]
   (c) Determine a single matrix of transformation that maps quadrilateral PQRS directly onto $P_2Q_2R_2S_2$.
   [Ans: $\begin{pmatrix} ^-1 & 0 \\ 0 & 1 \end{pmatrix}$]
6. (a) The image of triangle P with vertices (0, 0), (0, $^-2$), (2, 0) is P' with vertices (0, 0), (0, $^-4$), (4, 0) under transformation M. P' is then given a transformation N = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ to give P"
   (i) Find the transformation M **[Ans: M = $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$]**
   (ii) Describe M fully **[Ans: M is a matrix of enlargement at (0, 0) with scale factor 2]**
   (b) Find the co-ordinates of P" **[Ans: (0, 0) ($^-4$, 0) (0, 4)]**
   (c) Give a single matrix transformation which maps P onto P" **[Ans: $\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$]**
7. (a) A triangle PQR with vertices P ($^-2$, 1), Q(3, 1) and R(0, 3) is mapped onto triangle $P_1Q_1R_1$ whose vertices are P_1 ($^-6$, $^-1$), Q_1(9, 4) and R_1(0, 3) by a transformation described by matrix M. Triangle $P_1Q_1R_1$ is further transformed by a matrix N = $\begin{pmatrix} 1 & 0 \\ ^-1 & 3 \end{pmatrix}$ to triangle P_2, Q_2 R_2

Find;

(i) matrix M $\left[\textbf{Ans: M} = \begin{pmatrix} 3 & 0 \\ 1 & 1 \end{pmatrix}\right]$

(ii) the co-ordinates of $P_2Q_2R_2$ **[Ans: P_2 ($^-6$, 3) Q_2 (9, 3) R_2 (0, 9)]**

(b) Find the single matrix that would map PQR onto $P_2Q_2R_2$. Describe this transformation fully.

[Ans: $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ Matrix of enlargement at (0, 0) with scale factor 3]

(c) Find the area of;

(i) PQR **[Ans: 3 Sq.Units]**

(ii) $P_2 Q_2 R_2$ **[Ans: 27 Sq.Units]**

8. A transformation represented by the matrix $\begin{pmatrix} 4 & ^-2 \\ 3 & ^-2 \end{pmatrix}$ maps the vertices of triangle ABC on to its image vertices A′ (2, 1) B′ (10, 7) and C′(2, 0) respectively. The image of triangle ABC further undergoes another transformation represented by the matrix $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ to be Mapped on to triangle A" B" C" Find the:

(a) co-ordinates of the vertices A, B and C **[Ans: A(1, 1) B(3, 1) C(2, 3)]**

(b) co-ordinates of A" , B" and C" single matrix of the transformation which would map triangle A^{11} B^{11} C^{11} back on to triangle ABC $\left[\textbf{Ans: A"(4, 2) B"(20, 14) C"(4, 0) ; } \frac{1}{4}\begin{pmatrix} ^+2 & ^-2 \\ ^+3 & ^-4 \end{pmatrix}\right]$

9. Triangle ABC was transformed to its image triangle A′ B′ C′ by transformation T = $\begin{pmatrix} 6 & 3 \\ 4 & 3 \end{pmatrix}$. Find the area of triangleA′ B′ C′ if the area of triangle ABC is 12 cm^2. **[Ans: 72 cm^2]**

10. The points P(0, 2), Q(1, 4) and R(2, 2) are vertices of triangles PQR. The images of P, Q, and R under a reflection in the line x - y = 0 are P′,Q′ and R′ respectively. The points P′, Q′ and R′ are then mapped onto points P", Q" and R" respectively under an enlargement with centre (0, 0) and scale factor $^-2$.

(a) Write down the matrix for ;(i) reflection $\left[\textbf{Ans: } \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right]$ (ii) enlargement $\left[\textbf{Ans: } \begin{pmatrix} ^-2 & 0 \\ 0 & ^-2 \end{pmatrix}\right]$

(b) Hence determine the coordinates of the points

(i) P′, Q′ and R′ **[Ans: P′(2, 0) Q′(4, 1) R′(2, 2)]**

(ii) P", Q" and R" **[Ans: P"($^-4$, 0) Q"($^-8$, $^-2$) R"($^-4$, $^-4$)]**

11. The point A(5, 2) undergoes the transformation represented by M = $\begin{pmatrix} 3 & 2 \\ 1 & 0 \end{pmatrix}$ followed by a transformation T = $\begin{pmatrix} -6 \\ 11 \end{pmatrix}$. Determine the coordinates of the final image of A. **[Ans: A(13, 16)]**

12. (a) ABCD is a square with A(0, 5), B(0, 3), C(2, 3) and D(2, 5). It is rotated through 270^0 about the origin. Find the coordinates of A′, B′, C′ and D′, the images of A, B, C and D respectively.
[Ans: A′(5, 0) B′(3, 0) C′(3, $^-2$) D′(5, $^-2$)]

(b) Given that A′ B’C’D’ is then reflected onto A" B" C" D" in the line x - y = 0 find the coordinates of A", B", C" and D". **[Ans: A"(0, 5), B"(0, 3), C"($^-2$, 3) D"($^-2$, 5)]**

(c) Determine the matrix of a single transformation which would map A"B"C"D" back onto ABCD. Describe the matrix fully. **[Ans: $\begin{pmatrix} ^-1 & 0 \\ 0 & 1 \end{pmatrix}$ Matrix of reflection in the *y*-axis.]**

13. (a) Point B(-2, 3) was mapped onto its image point B^1 under a reflection in $y = x$. Find the coordinates of B′. **[Ans: B′(3, $^-2$)]**

(b) The transformation described by the matrix $\begin{pmatrix} 3 & x \\ y & 3 \end{pmatrix}$ maps point A(3, 5) onto point A′(4, 9). Find the values of x and y. **[Ans: $x = {^-1}$, $y = {^-2}$]**

14. (a) Triangle ABC with A(7, 8), B(1, 9) and C(3, 5) is reflected in the line x - y = 0 to give its image triangle A′ B′ C′. Triangle A′ B′ C′ is then given a positive quarter turn to form its image triangle A"B"C" about centre O(0, 0). Find;

(i) A′, B′ and C′ **[Ans: A′(8, 7) B′(9, 1) C′(5, 3)]** (ii) A", B" and C" **[Ans: A"($^-7$, 8) B"($^-1$, 9) C"($^-3$, 5)]**

(b) Show triangle ABC and its images on the same axes.

15. Triangle PQR with area 15 cm^2 was transformed to its image triangle P′ Q′ R by a transformation matrix $N = \begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix}$. Find the area of triangle P′ Q′ R′. **[Ans: 105 cm^2]**

16. The vertices of triangle PQR, P(0, 1), Q(2, 1) and R(2, 5) are mapped onto the triangle P′ Q′ R′ by the transformation whose matrix is $\begin{pmatrix} 4 & 0 \\ -2 & 1 \end{pmatrix}$.
    (a) Find the;
        (i) Coordinates of the vertices of the image triangle P′ Q′ R′ **[Ans: P′(0, 1) Q′(8, -3) R′(8, 1)]**
        (ii) Ratio of the area of triangle PQR to area of triangle P′ Q′ R′ **[Ans: PQR : P′Q′R′ = 1 : 4]**
    (b) Plot on the same axes triangle PQR and its image P′ Q′ R′
    (c) Determine the matrix of transformation which maps P′ Q′ R′ back onto PQR. **[Ans: $\frac{1}{4}\begin{pmatrix} 1 & 0 \\ 2 & 4 \end{pmatrix}$]**

17. The vertices of triangle P are (1, 1), (2, 3) and (4, 3). The matrix $R = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$ maps P onto P′ and the matrix $E = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ maps P′ onto P".
    (a) Draw the three triangles P, P′ and P" on the same diagram
    (b) What is the image of P" under the transformation described by RE^{-1}? **[Ans: $(\frac{1}{4}, \frac{1}{4})$ $(\frac{1}{2}, \frac{3}{4})$ $(1, \frac{3}{4})$]**
    (c) What is the image of P under the transformation described by ER^{-1}? **[Ans:(2, 2) (6, 4) (6, 8)]**

18. A transformation represented by the matrix $N = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ maps the points A(0, 0), B(2, 0), C(2, 3) and D(0, 3) of quadrilateral ABCD onto A′ B′ C′ D′.
    (a) Draw the quadrilateral ABCD and its image A′ B′ C′ D′.
    (b) Hence or otherwise determine the area of A′ B′ C′ D′. **[Ans: 18 Sq.units]**
    (c) A transformation represented by matrix $M = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ maps A′ B′ C′ D′ onto A"B"C"D". Find the coordinates A", B", C" and D". Hence draw A"B"C"D" on the same graph.
    [Ans: A"(0, 0) B"($^-2$, $^-4$) C"($^-8$, $^-7$) D"($^-6$, $^-3$)]
    (d) Determine the single matrix which maps A"B"C"D" back to ABCD. **[Ans: $\frac{1}{4}\begin{pmatrix} 1 & ^-2 \\ ^-2 & 1 \end{pmatrix}$]**

19. A transformation T_1 is defined by a reflection in the line $y = x$ followed by a half turn about the origin. Find the matrix of transformation T_1. **[Ans: $T_1 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$]**

20. The points A($^-6$, 5), B(3, 2) and C(3, $^-1$) are vertices of triangle ABC. Triangle ABC undergoes a transformation represented by matrix $M = \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$ to form image triangle A′B′C′.Triangle A′B′C′ is then reflected in the x-axis to form the image triangle A"B"C".
    (a) Find the coordinates of;
        (i) A′, B′ and C′. **[Ans: A′($^-6$, 3) B′(3, 12) C′(3, 3)]**
        (ii) A", B" and C" **[Ans: A"($^-6$, $^-3$) B"(3, $^-12$) C"(3, $^-3$)]**
    (b) On the same graph, show triangle ABC and its images.
    (c) Calculate the area of triangle A′B′C′, the image of ABC and hence calculate the area of triangle ABC. **[Ans: Area A′B′C′ = 40.5 sq.units. Area ABC = 13.5 sq.units]**

21. Under an enlargement scale factor 3, a point P is mapped onto its image P′(18, $^-15$), find the coordinates of P. **[Ans: P(6, $^-5$)]**

22. The transformations T and S are defined as follows; T = reflection in the line $y = x$. S = positive quarter turn about the origin. The points A(3, 7), B(3, 4) andC($^-1$, 4) are the vertices of triangle ABC whose image under T is triangle A′B′C′. Triangle A"B"C" is the image of triangle A′B′C′ under S.
    (a) Find the coordinates of;

(i) A′, B′ and C′ **[Ans: A′(7, 3) B′(4, 3) C′(4, ⁻1)]**

(ii) A", B" and C" **[Ans: A"(⁻3, 7) B"(⁻3, 4) C"(1, 4)]**

(b) Show triangle ABC and its images on the same graph.

(c) Transformation T followed by S can be replaced by a single transformation P. Find the matrix P.
[Ans: P = $\begin{pmatrix} ^{-}1 & 0 \\ 0 & 1 \end{pmatrix}$**]**

23. A transformation represented by the matrix M = $\begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}$ maps the points A(0, 0), B(2, 0), C(2, 3) and D(0, 3) of the quadrilateral ABCD onto A′B′C′D′. Another transformation N = $\begin{pmatrix} 0 & ^{-}1 \\ ^{-}1 & 0 \end{pmatrix}$ maps A′B′C′D′ onto A"B"C"D".

(a) State the coordinates of A′, B′, C′ and D′ the images of A, B, C and D and the coordinates of A", B", C" and D" the images of A′, B′, C′ and D′respectively. **[Ans: A′(0, 0) B′(4, 2) C′(7, ⁻4) D′(6, ⁻3); A"(0, 0) B"(⁻2, ⁻4) C"(4, ⁻7) D"(6, ⁻3)]**

(b) Draw the quadrilateral ABCD and its images on the same graph.

(c) Hence or other wise determine the area of A′ B′ C′ D′ **[Ans: 18 sq.units]**

(d) Determine the single matrix which maps A"B"C"D" back to ABCD. **[Ans:** $\frac{1}{5}\begin{pmatrix} ^{-}1 & ^{-}2 \\ 2 & ^{-}1 \end{pmatrix}$**]**

24. (a) Triangle A′B′C′ is the image of triangle ABC under transformation M = $\begin{pmatrix} 2 & 1 \\ 2 & 5 \end{pmatrix}$. Find the area of ABC if the area of A′B′C′ is 28 cm^2. **[Ans: 3.5 cm^2]**

(b) Point A(0, 3) is reflected in the line $y + x = 0$. Find the coordinates of its image A′. **[Ans: (⁻3, 0)]**

25. (a) A unit square whose vertices O(0, 0), A(1, 0), B(0, 1) and C(1, 1) is transformed by rotating through a positive quarter turn about the origin. Find the matrix for this transformation. **[Ans:** $\begin{pmatrix} 0 & ^{-}1 \\ 1 & 0 \end{pmatrix}$**]**

(b) Given T = $\begin{pmatrix} 0 & ^{-}1 \\ 1 & 0 \end{pmatrix}$ and M = $\begin{pmatrix} ^{-}1 & 0 \\ 0 & ^{-}1 \end{pmatrix}$, find;

(i) the images of the points A(0, 3) and B(5, 3) under the transformation TM. **[Ans: A′(3, 0) B′(3, ⁻5)]**

(ii) the matrix of transformation which will map the images of A and B back to their original positions.
[Ans: $\begin{pmatrix} 0 & ^{-}1 \\ 1 & 0 \end{pmatrix}$**]**

26. (a) A transformation represented by the matrix M = $\begin{pmatrix} 1 & ^{-}1 \\ 4 & 8 \end{pmatrix}$ maps triangle ABC whose area is 24.5 cm^2 onto A′B′C′. Find the area of triangle A′B′C′. **[Ans: 294 cm^2]**

27. Rectangle OABC is transformed into a parallelogram O′ A′ B′ C′ by the matrix $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ where O′(0, 0), A′(6, 3), B′(8,7), C′(2, 4). Find the;

(a) matrix which maps the parallegram O′ A′ B′ C′ back onto the rectangle OABC. **[Ans:** $\frac{1}{3}\begin{pmatrix} 2 & ^{-}1 \\ ^{-}1 & 2 \end{pmatrix}$**]**

(b) Coordinates of O, A, B and C. **[Ans: O(0, 0) A(3, 0) B(3, 2) C(0, 2)]**

(c) Area of the rectangle OABC and hence the area of the parallegram O′ A′ B′ C′.
[OABC = 6 sq.units; O′A′B′C′ = 18 sq.units]

28. The points A(1, 0), B(3, 0), C(3, 1), D(1, 1) are vertices of a rectangle ABCD. The images of A, B, C and D under a transformation T = $\begin{pmatrix} ^{-}2 & 0 \\ 0 & -2 \end{pmatrix}$ are A′, B′, C′ and D′ respectively. The images A′, B′, C′ and D′ are then mapped onto the points A", B", C" and D" respectively under a transformation M = $\begin{pmatrix} 0 & ^{-}1 \\ ^{-}1 & 0 \end{pmatrix}$.

(a) Determine the coordinates of the points

(i) A′, B′, C′ and D′ **[Ans: A′(⁻2, 0) B′(⁻6,0) C′(⁻6, ⁻2) D′(⁻2, ⁻2)]**

(ii) A", B", C" and D" **[Ans: A"(0, 2) B"(0, 6) C"(2, 6) D"(2, 2)]**

(b) Find a single matrix of transformation which would map rectangle A"B"C"D" back onto ABCD. **[Ans: $\begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$]**

29. The point A(-2, 1), B(1, 4) and C(1, 1) are vertices of a triangle ABC. The points A′, B′ and C′ are the images of A, B and C under a transformation described by matrix M = $\begin{pmatrix} 3 & 0 \\ -2 & 1 \end{pmatrix}$. The points A", B" and C" are the images of A′, B′ and C′ under a transformation described by matrix N = $\begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$.
(a) Find the coordinates of; (i) A′, B′ and C′ **[Ans: A′(⁻6, 5) B′(3, 2) C′(3, ⁻1)]**
(ii) A", B" and C" **[Ans: A"(⁻6, 3) B"(3, 12) C"(3,3)]**
(b) Find the ratio of areas of triangles ABC to A"B"C". **[Ans: ABC: A"B"C" = 1 : 9]**
(c) Describe a single matrix that maps triangle ABC onto A"B"C"
[Ans: $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ A matrix of enlargement at the centre (0, 0) with scale factor 3]

30. (a) The vertices of a triangle RPQ are (1, 3) (1, 4) and (3, 3) respectively. Find the coordinates of the image triangle $R_1P_1Q_1$ under the reflection in the line $x - y = 0$. **[Ans: R_1(3, 1) P_1(4, 1) Q_1(3, 3)]**
(b) The image $R_1P_1Q_1$ is then mapped onto $R_2P_2Q_2$ by a positive $\frac{3}{4}$ turn about the origin. Find the coordinates of triangle R_2 P_2 Q_2. **[Ans: R2(1, ⁻3) P2(1, ⁻4) Q2(3, ⁻3)]**
(c) Find a single matrix of transformation that would map $R_2P_2Q_2$ back to RPQ. **[Ans: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$]**

31. (a) ABC is a triangle with vertices A(⁻1, 1), B(4,1) and C(1, 5).Triangle ABC is mapped onto triangle A′B′C′ by a transformation represented by T = $\begin{pmatrix} ^-2 & ^-1 \\ ^-1 & 2 \end{pmatrix}$. Find the coordinates of the vertices of triangle A′B′C′.
[Ans: A′(1, 3) B′(⁻9, ⁻2) C′(⁻7, 9)]
(b) Triangle A′B′C′ is then mapped onto triangle A"B"C" by a transformation matrix K = $\begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}$. Find the coordinates of the vertices of triangle A"B"C". **[Ans: A"(5, ⁻5) B"(⁻20, ⁻5) C"(⁻5, ⁻25)]**
(c) Find a single transformation matrix which would map triangle A"B"C" on to triangle ABC. Hence describe the transformation fully. **[Ans: $\begin{pmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{5} \end{pmatrix}$ matrix of enlargement at the centre (0, 0) with scale factor $\frac{1}{5}$]**

32. (a) An object of area 10.5 cm^2 is mapped on to its image of area 105 cm^3 by a matrix of transformation given by, $\begin{pmatrix} 1 & ^-1 \\ m & 4 \end{pmatrix}$. Determine the value of m. **[Ans: m = 6]**
(b) If Q(⁻2, 5) is the image of P under a negative quarter turn about the origin, find the coordinates of P.
[Ans: P(⁻5, ⁻2)]
(c) The image of P(6, 3) after a reflection is P′(3, 6).
(i) Plot the points P and P′ on a graph paper.
(ii) Construct the line of reflection. Hence find the equation of the line of reflection. **[Ans: $y = x$]**
(d) An object at (0, 0) undergoes a translation A = $\begin{pmatrix} 5 \\ -12 \end{pmatrix}$ then followed by translation B = $\begin{pmatrix} 3 \\ 6 \end{pmatrix}$
(i) Find a single translation equivalent to the two translations A and B. **[Ans: T = $\begin{pmatrix} 8 \\ -6 \end{pmatrix}$]**
(ii) How far is the object from (0, 0)? **[Ans: 10 units]**

33. Square A(⁻2, ⁻1), B(1, ⁻1), C(1, ⁻4), D(⁻2, ⁻4) is enlarged to square A′(4, 1), B′(3, 1), C′(3, 2), D′(4, 2)
(a) Choose a suitable scale and draw these squares on a graph paper, labelling the vertices carefully
(b) Hence find the scale factor and the coordinates of the centre of enlargement

34. (a) If A(2, 4) is mapped onto A′(x, y) by a reflection in the line $y = x – 4$. Find the coordinates of A′
[Ans: A′(8, ⁻2)]
(b) Triangle ABC with vertices A(1, 2), B(2, 5) and C(3, 3) is mapped onto the triangle A′ B′ C′ with vertices A′(3, 0), B′(6, 1) and C′(4, 2) by a reflection. Draw both triangles on the same graph and find the equation of the mirror line. **[Ans: $y = x − 1$]**

LINEAR PROGRAMING AND INNEQUALITY

RECALL

Graphs for inequalities

◊ If the inequality is $\geq$ or $\leq$, i.e the points on the boundary line are included in the region representing the inequality. The boundary line is drawn as a continuous line, ________

◊ If the inequality is $>$ or $<$, the points on the boundary line are not included in the region representing the inequality. The boundary line is drawn as a broken line (……………….)

◊ Find the unwanted region out of the two by choosing a test point on the side of the boundary line. The co-ordinates of the test point are substituted in the inequality a test is performed to find out whether the point satisfies the inequality.

◊ Shade the unwanted region.

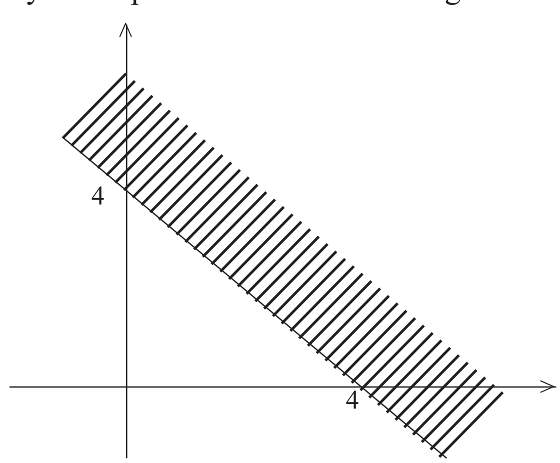
Linear programming

◊ The following are some of the words and their corresponding symbols as used in linear programming.

Doesn't exceed $\leq$	Must transport $\geq$	Exceeds $>$
A maximum of $\leq$	Wishes to transport $\geq$	More than $>$
At least $\geq$	Contracted $\geq$	Less than $<$.

TRIAL QUESTIONS

1. By shading the unwanted region on the same axes show the region satisfying the inequalities; $y \leq 4x + 12$, $3y + x > {}^-3$ and $x \leq 2$
2. By shading the unwanted regions show the region which satisfies the inequalities.
   $x + y \leq 3$, $y \geq x - 4$, $y + 7x > {}^-4$
   Find the area of the wanted region
3. Find the inequality that represents the unshaded region in the diagram below.



4. (a) By shading the unwanted regions show on a graph using a scale of 2 *cm*: 1 unit on both axes the region R which satisfies all the 3 inequalities below:
   (i) $3x + 5y \geq 15$ (ii) $x < y$ (iii) $y \leq 5$
   (b) Study the graph below and use it to complete the inequalities below which satisfy the unshaded region R.

5. The manager of a cinema wishes to divide the seats available into two classes A and B. There are not more than 120 seats available. There must be at least twice as many B class as there are A class seats. Class A seats are priced at 1500 shs each and class B at 1000 shs each. At least 100,000/= should be collected at each show to meet the expenses. Taking x as the number of class A seats sold and y as the number of class B seats sold, write down the inequalities representing the above information and plot them on a graph. From the graph;
   (a) (i) Find the number of seats of each kind which must be sold to give the maximum profit
   (ii) State the maximum profit.
   (b) Find the least number of seats that must be sold in order to incur no loss.
6. Solve the inequality and show the solution on a number line $^{-}8 < \frac{3}{4}x - 2 \leq x - 3$
7. A certain school wishes to take its S4 students for a picnic at a national park. The school has hired a bus and a mini-bus to take the students. Each trip of the bus costs sh 80,000 and that of the mini-bus shs 50,000. The bus has a capacity of 57 students and the mini-bus 19 students. All the 171 students in S4 contributed a total of shs 400,000 for hiring the vehicles and must all go for the picnic. The mini-bus must make more trips than the bus because it travels faster. If x and y represent the number of trips made by the bus and mini-bus respectively;
   (a) Write down five inequalities representing the above information
   (b) Plot these inequalities on the same axes.
   (c) By shading the unwanted region, show the region satisfying all the above inequalities
   (d) Find the number of trips each vehicle should make so as to spend the least amount of money. Hence find the amount of money saved.
8. A manufacturer makes two types of drinks A and B. The following conditions apply to daily production.
   (i) Each of type A costs shs 3,000 and each of type B costs shs 5,000 and the manufacturer has a maximum of shs 450,000 available.
   (ii) Due to labour shortage, the production of type A plus four times that of B should not exceed 160.
   (iii) A market study recommended that the number of type B produced should not exceed twice the number of type A produced.
   (a) Given that x drinks of type A and y drinks of type B are made, write down three inequalities apart from $x \geq 0$, $y \geq 0$ satisfying the above conditions.
   (b) Show graphically the region containing the points satisfying the above conditions.
   (c) Taking $x + 2y$ as a suitable expression for the manufacturer's profit, find the number of each type of drink that should be made to obtained the greatest profit.
9. (a) On the same axes, show the region satisfying the following inequalities by shading out the unwanted regions; $y \leq 4$, $y + 1 \geq 0$, $x + y \leq 6$ and $x - y + 8 \geq 0$.
   (b) Calculate the area of the region so formed.
10. Given that P = $\{(x, y);\, 2x - 3y \leq 6\}$ and Q = $\{(x, y);\, x + y < 6\}$ show by shading the unwanted regions, the region representing $P \cap Q$
11. Karibu Hotel has 7 roasters of 200 *kg* oven capacity and 6 roasters of 400 *kg* oven capacity. The 200 *kg* oven capacity roaster can be used 5 times a day. The 400 *kg* oven capacity roaster can be used only 2 times a day. Each roaster must be operated by only one chef. On a given Saturday the Hotel is contracted to roast 9000 *kg* of meat for guests at a wedding ceremony. On that day, only 11 chefs were available. The 200 *kg* oven capacity roasters each needs shs, 12,000 per day to run and the 400 *kg* oven capacity roasters each needs shs. 20,000 per day to run. If x and y represent the number of 200 *kg* oven capacity roasters and 400 *kg* oven capacity roasters to be used respectively by the Hotel.
   (a) Write down six inequalities representing the above information.
   (b) Plot on the same axes, graphs for the inequalities, shading out the unwanted region.
   (c) Use your graph to find the number of each type of roaster the Hotel should use so as to minimise costs.

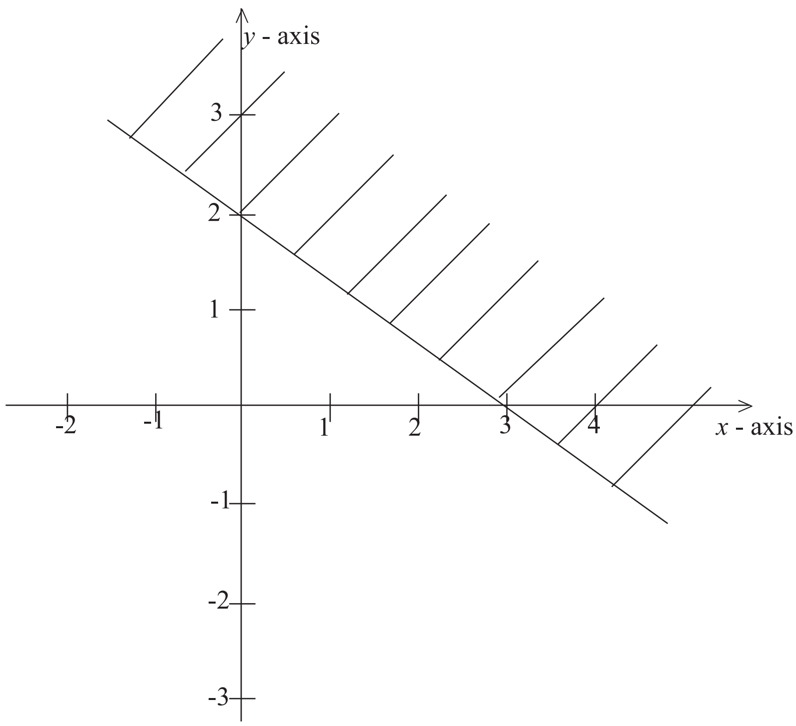
12. The profits for two types of chicken, broilers x and layers y, produced by NICO poultry farm are shs 100,000 and shs 150,000 respectively. Broilers require 40 *kg* of starter meal and 25 *kg* of smash meal. Layers require 20 *kg* of starter meal and 30 *kg* of smash meal. Given that there are 120 *kg* of starter meal and 150 *kg* of smash meal available.

    (i) Use inequalities to represent this information

    (ii) Represent the inequalities above on a graph and use it to find the possible number of chicken of each type that can be kept.

    (iii) Calculate the maximum profit.

13. State the inequality that satisfies the unwanted region shown below.



14. Solve the inequality $3x - 2 < 10 + x < 2 + 5x$. Show the solution on a number line.

15. A workshop makes two types of chairs: Rockers and Swivels. Two operations A and B are used. Operation A is limited to 20 hrs a day. Operation B is limited to 15 hrs. a day. The table below shows the time each operation takes for a chair and the profit made.

Chair	Operation A	Operation B	Profit
Rocker	2 hrs	3 hrs	\$12
Swivel	4 hrs	1 hr	\$10

    (a) Write down all the inequalities that satisfy these information.

    (b) Represent these inequalities on a graph.

    (c) Find the maximum profit.

    (d) Find how many chairs of each type should be made to maximize the profit.

16. A school intends to transport 900 students to a field trip using a school bus and van. The bus can carry a maximum of 60 students while a van can carry a maximum of 45 students. The van has to make more than twice the number of trips made by the bus. The total number of trips has to be less than 20. The cost per trip is shs. 100,000 and shs. 80,000 per bus and a van respectively.

    (a) Taking x and y to represent the number of trips by a bus and a van respectively. Write down five inequalities representing above information.

    (b) Use a scale of 4 *cm*: 5 units on both axes. Graph these inequalities and shade out the unwanted regions.

    (c) List down all the possible trips made by the vehicles.

    (d) Determine the maximum expenditure incurred by the school

17. (a) Using a number line, illustrate the solution set for the inequality $^-4 \leq x < 1$

(b) Dot service wants to transport 400 workers to a working centre using x small vans of capacity 40 persons each and y large buses of capacity 80 persons each. The transport department has a maximum of 89 drivers on duty and the largest number of buses is 4.

(i) Write down inequalities representing the restrictions on the values of x and y.

(ii) Plot on the same axes the inequalities, shading out the unwanted region.

(iii) If the expenses of running each small van are shs.200,000 and each large bus are shs.300,000. Determine the maximum cost.

18. A farmer has x goats and y cows. The food cost for each goat is shs.800 and for each cow is shs.1600 per day. Only shs.14,400 is available for animal food. There is no room for more than 14 animals. There must be at least 9 goats and at least 3 cows.

(a) Write down all the inequalities that represent the above information.

(b) Represent the inequalities on a graph by shading the unwanted regions.

(c) From your graph, find the number of goats and cows that must be kept so that the food cost is to be minimum.

19. A tailor has shs.180,000 to spend on materials for making shirts and trousers in a week. The material for a shirt cost him shs.12,000 and for a pair of trousers shs.15,000, the time taken to make a shirt is $2\frac{1}{2}$ hours and a pair of trousers 5 hours but he works at most 40 hours per week. He needs to make at least as many shirts as pairs of trousers. He makes a profit of shs.2000 on each shirt and shs.5000 on each pair of trousers. If x and y represents shirts and pair of trousers respectively,

(a) Write down all the inequalities for the above information

(b) Plot a graph of the inequalities, shading out the unwanted regions.

(c) How many of each should he make in a week to maximize his profit?

20. A shop keeper orders bags of rice. The cost price of a large bag is shs.180,000 and that of small of a small bag is shs.80,000. She is prepared to spend up to shs.4,000,000 altogether and needs twice as many large bags as small bags, with a minimum of 10 large and 20 small bags.

(a) What is the greatest number of bags she can buy?

(b) The profit is shs.20,000 on a large bag and shs.10,000 on small bag.

21. A school has 70 hectares or land available for growing maize and beans. The cost per hectare for growing maize and beans is shs.90,000 and shs.60,000 respectively, and it has only shs.5,400,000 available. The labor per hectare is 2 man days for the maize and 4 man days for the beans, and a total of 240 man days of labor are available. If x and y represent the number of hectares to be used for maize and beans respectively, write down in their simplest form the inequalities which x and y must satisfy.

(b) Using a scale of 1 *cm*: 5 units on both axes, draw the appropriate straight lines to find the region in which the point (x, y) must lie if the inequalities are to be satisfied.

22. Kampala city council has an area of 40,000 m^2 set aside for a modern car taxi park. The average area for parking a car is 800 m^2 while the area for parking a minibus is 2000 m^2. Not more than 30 vehicles can be accommodated. The parking charge for the car is shs.1500 and the charge for a minibus is shs.2500.

(a) How many of each should be parked for maximum income.

(b) State the maximum income.